ESTIMATION OF THE VOLATILITY PROCESS OF SPDE

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Abstract. Many phenomena in physics or finance can be well described by stochastic partial differential equations of the form

$$dX_t = (\alpha_1 \partial_{xx} + \alpha_2 \partial_x + \alpha_3) X_t dt + \sigma_t dW_t,$$

where $\alpha_1, \alpha_2, \alpha_3$ are real-valued fixed parameters and σ is a Hilbert-Schmidt operator-valued volatility process. Based on discrete observations of the solution of such an SPDE, we introduce feasible ways to make inference on the volatility process σ via *semigroup adjusted multipower variations*.

Our method circumvents a delicate and overlooked problem in data analyses of such systems: Due to their inherent infinite-dimensionality, it is desirable to reduce the complexity of the models by projecting them down onto some finite-dimensional linear subspace in which most of the variation is captured, that is, conducting a (functional) principal component analysis. However, an ad-hoc application of this classical technique produces models that are'nt solutions to an SPDE of the type above anymore. This may induce undesired properties such as arbitrage opportunities in term structure models.

Instead of reducing the dimensionality of the process directly, we suppose to refrain from making restricting assumptions on the process a priori and to reduce the dimensionality of the volatility process instead.

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