MINI PROJECTS MEASURE THEORETIC PROBABILITY

Project 1 – presentation date: September 21st.

Part 1: Recall the definition of ordinals, explain what limit ordinals are. Recall Axiom of Choice and Zorn’s Lemma (mention equivalence, do not prove it). Explain why by Zorn’s lemma the set of countable ordinals is well-defined.

Part 2: provide the proof of Proposition 1.4.

Project 2 – presentation date: September 28th.

Two-person version:

Part 1: Definition 2.1 and Lemma 2.4 with proof.

Part 2: Definition 2.2, Theorem 2.3 without proof, Lemma 2.5 with proof Theorem 2.6 with proof.

Three-person version:

Part 1: Definition 2.1 and Lemma 2.4 with proof.

Part 2: Definition 2.2, Theorem 2.3 without proof but with a rough sketch of what is to be done, Lemma 2.5 with proof Theorem 2.6 with proof.

Part 3: (Optional): Theorem 2.6 with proof, and explain why the collection of Lebesgue-measurable sets has the same cardinality as $2^\mathbb{R}$.

Project 3 – presentation date: October 5th.

NB: Part 1 and 2 below can be done independently.

Part 1: Theorem 3.1 with proof, mention also the results of Exercise 3.7 (without proof).

Part 2: Exercise 3.11 + solution. Send your solution to the TA before Monday, October 3rd, 23:59 CET so she can give feed-back.

Project 4 – presentation date: October 12th.

Part 1: Proof of Fatou’s lemma (Lemma 4.15), Definition 4.17, Proposition 4.18 (i.e., Exercise 4.3).


Part 3: (Optional) Proof of Scheffé’s lemma (Lemma 4.20) Example 4.21.

Project 5 – presentation date: October 19th.

Section 4.5: functions of bounded variation and Stieltjes integrals.
Project 6 – presentation date: October 26th

NB: Part 1, 2 and 3 below can be done independently.

Part 1: Exercise 5.10.
Part 2: Exercise 5.11.

Please send your solution to the TA before Monday, October 3rd, 23:59 CET so she can give feedback.

Project 7 – presentation date: November 2nd

Section 8.2:

Part 2: Theorem 8.14 with proof (perhaps a sketch of the proof, time yourself and make sure you need no more than 12 minutes).
Part 3: Proposition 8.15. Please also explicitly mention the case $h(x) = x, x \in \mathbb{R}$.

Project 8 – presentation date: November 9th

Part 2: Exercise 7.10.
Part 3: Exercise 7.11. (This exercise is independent of exercises 7.9 and 7.11.)

Please send your solution to the TA before Monday, November 14th, 23:59 CET so she can give feedback.

Project 9 – presentation date: November 16th

Part 3: Theorem 10.15 and Corollary 10.16. Remark: in the proof of Theorem 10.15 and 10.16 you can use Theorem 10.5 instead of Theorem 10.8 (Theorem 10.5 was already discussed in class.)

Project 10 – presentation date: November 23rd

Remark: you will need Section 10.3 (i.e., the project 9) to solve these exercises.


Please send your solution to the TA before Monday, November 28th, 23:59 CET so she can give feedback.
MINI PROJECTS MEASURE THEORETIC PROBABILITY

Project 11 – presentation date: November 30\textsuperscript{th}

This project involves an alternative proof to the Radon-Nikodym theorem based on martingale convergence results. See e.g. planetmath.org/node/41293/pdf. If you use this document as a starting point, a logical set-up would be:

Part 1: Give the statement of the Radon-Nikodym theorem your team aims to prove, and give a proof to Lemma 1 in the document.

Part 2: Lemma 2 in the document.

Part 3: Complete the proof of the Radon-Nikodym theorem.

Project 12 – presentation date: December 7\textsuperscript{th}

Part 1: Lemma 13.16 and exercise 13.3

Part 2: Alternative proof of Theorem 12.16 Part 1: upto and including equation (13.11)


Project 13 – presentation date: December 14\textsuperscript{th}

Last year’s exams (details will follow).