MTP 2014 HOMEWORK SERIES 3

(DEADLINE OCT. 8TH 2014, 10:15 AM)

HOMEWORK

(i) Read Theorem 4.12 and its proof.
(ii) Observe that in the definition of an integral (Definition 4.8 in the lecture notes) we do not actually use that \( f \) is measurable (this definition would work for an arbitrary mapping \( f : \mathbb{R} \to \mathbb{R} \)). However, in order for the integral to have ‘good’ properties measurability is necessary. To understand this, explain where in the proof of Theorem 4.12 it is used that \( f_n, n \in \mathbb{N} \), is measurable. Does this imply that \( f \) is measurable?
(iii) Solve exercise 4.2.

PRESENTATIONS

**Presentation 1:** Proof of Theorem 4.12 for the case that \( f_{n+1}(s) \geq f_n(s) \) for all \( s \in S \).

**Presentation 2:** Proof of Theorem 4.12 for the case \( f_{n+1}(s) \geq f_n(s) \) for a.e. \( s \in S \), based on Presentation 1. Also explain where measurability was used.

**Presentation 3:** Exercise 4.2.