**Errata and comments to the paper "Clebsch-Gordan coefficients for $SU(2)$ and Hahn polynomials" by T. H. Koornwinder, Nieuw Arch. Wisk. (3) 29 (1981), 140–155**

*Last modified: April 29, 2025*

**p.140, footnote**: This statement of R. Askey is on p.8 of the paper

G. Mackie, with an addendum by R. Askey, *Arthur Erdélyi*, Applicable Anal. **8** (1978), 1–10.

The full statement reads:

"One very important application of some of these discrete orthogonal (Hahn) polynomials was known over thirty years ago, but it was only recognized within the last two years. In studying quantum theory of angular momentum (and some other problems as well) physicists computed the coefficients that occur in the decomposition of the tensor product of two representations of SU(2). These are called Clebsch–Gordon coefficients or 3-$j$ symbols. They have two orthogonality relations. One is a disguised version of the orthogonality of Hahn polynomials (which had been found by Tchebychef[1]: the other is a disguised version of an orthogonality relation for polynomials that are called dual Hahn polynomials. Unfortunately no one was aware of this, neither the physicists who discovered the group theoretic setting for these results nor the mathematicians who considered the orthogonal polynomials, so this is not mentioned in H.T.F."

**p.146, (3.9)**: More precisely, fix $\ell_1, \ell_2$, and $j$ such that $|j| \le \ell_1 + \ell_2$. The functions $\phi_j^{\ell_1,\ell_2,\ell}$ $(|\ell_1 - \ell_2| \vee |j| \le \ell \le \ell_1 + \ell_2)$ and $\phi_{j_1}^{\ell_1} \otimes \phi_{j-j_1}^{\ell_2}$ $((-\ell_1) \vee (j - \ell_2) \le j_1 \le \ell_1 \wedge (j + \ell_2))$ form two orthonormal bases of the same real vector space of dimension

$$\min(2\ell_1, 2\ell_2, \ell_1 + \ell_2 + j, \ell_1 + \ell_2 - j) + 1.$$

The two bases are connected by

$$\phi_j^{\ell_1,\ell_2,\ell} = \sum_{j_1=(-\ell_1)\vee(j-\ell_2)}^{\ell_1\wedge(j+\ell_2)} C_{j_1,j-j_1,j}^{\ell_1,\ell_2,\ell}\, \phi_{j_1}^{\ell_1} \otimes \phi_{j-j_1}^{\ell_2}$$

and

$$\phi_{j_1}^{\ell_1} \otimes \phi_{j-j_1}^{\ell_2} = \sum_{\ell=|\ell_1-\ell_2|\vee|j|}^{\ell_1+\ell_2} C_{j_1,j-j_1,j}^{\ell_1,\ell_2,\ell}\, \phi_j^{\ell_1,\ell_2,\ell}.$$

**p.147, (3.13)**: In the middle part of the first line of (3.13) add a third argument $\ell$ to the upper index.

**p.151, (4.3)**: On the right-hand side replace in the numerator of the summand $(-x)_x$ by $(-x)_k$.

**p.151, fifth line after (4.3)**: Replace "$x = \ell_1 - \ell_2$" by "$x = \ell_1 - j_1$".

**p.155, [8]**: Replace 1982 (the year of publication) by 1928.

[1] P. Tchebychef, *On the interpolation of equidistant values*, Zapiski Imperatorskoi Akademii Nauk **25** (1875), paper no. 5; translated in French: *Sur l'interpolation des valeurs equidistantes*, in: *Oeuvres de P. L. Tchebychef* (A. Markoff and N. Sonin, eds.), St. Petersburg, 1899/1907), Vol. 2, pp. 217–224.

**Further notes** Dunkl gives in §4.1 of the paper

C. F. Dunkl, *Spherical functions on compact groups and applications to special functions*, in: Symposia Mathematica, Vol. XXII, Istituto Nazionale di Alta Matematica, 1977, pp. 145–161.

an interpretation of Hahn polynomials as intertwining functions on the symmetric group $S_N$ with respect to the subgroups $S_s \times S_{N-s}$ and $S_t \times S_{N-t}$. Scarabotti gives in §4.2 of the paper

F. Scarabotti, *Multidimensional Hahn polynomials, intertwining functions on the symmetric group and Clebsch-Gordan coefficients*, Methods Appl. Anal. **14** (2007), 355–386.

a connection between the two interpretations of Hahn polynomials as Clebsch–Gordan coefficients and as intertwining functions.