On the paper "Jacobi polynomial expansions of Jacobi polynomials with non-negative coefficients" by R. Askey and G. Gasper

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This note deals with the paper

R. Askey and G. Gasper, *Jacobi polynomial expansions of Jacobi polynomials with non-negative coefficients*, Proc. Cambridge Philos. Soc. 70 (1971), 243–255; MR0296369.

In (1.6) the term $-(a-2)(a+b)$ should read $-(\alpha-2)(a+b)$. This follows from the expression for $N(\alpha,\alpha)$ on p.251 together with substitution of $\beta=\alpha$ in the expression for H_0 on p.249.

So the statement is that for $-1<\alpha\le 0$ we have

$$P_n^{(a,b)}(x)=\sum_{k=0}^n g(n,k)\,P_k^{(\alpha,\alpha)}(x)\quad\text{with } g(n,k)\ge 0$$

iff (always assuming $a,b>-1$) we have $a\ge b$ and

$$(\alpha+2)(a^2+b^2)-2(\alpha+1)ab-(\alpha-2)(a+b)-4\alpha\ge 0.$$

Thus, for given $\alpha\in(-1,0]$ nonnegativity of the $g(n,k)$ holds iff $-1<b\le a$ while (a,b) is not in the interior of the ellipse with long axis from $(-2,-2)$ to (α,α) and short axis from

$$\Big(\tfrac12\alpha-1+\frac{\tfrac12\alpha+1}{\sqrt{2\alpha+3}},\tfrac12\alpha-1-\frac{\tfrac12\alpha+1}{\sqrt{2\alpha+3}}\Big)\quad\text{to}\quad\Big(\tfrac12\alpha-1-\frac{\tfrac12\alpha+1}{\sqrt{2\alpha+3}},\tfrac12\alpha-1+\frac{\tfrac12\alpha+1}{\sqrt{2\alpha+3}}\Big).$$

A few further errata communicated to me by George Gasper:

- p.245, line 12: $a+b$ should read $a-b$
- p.252, line 6: the a coefficient in the first sum should read a_n
- p.254, line 13: put 0 (zero) as the missing lower limit on the integral sign