Errata and comments on the book *Basic hypergeometric series*, Second edition, by G. Gasper and M. Rahman

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These are errata and comments on the book

G. Gasper and M. Rahman, *Basic hypergeometric series*, Cambridge University Press, Second ed., 2004, ISBN 9780521833578.

p. 101, Exercise 3.2(iii):
We can combine the two equalities in (i) and (ii) as

$$ {}_3\phi_2\left(\begin{matrix} a,b,-b \\ b^2,-az \end{matrix};q,z\right) = \frac{(-z;q)_\infty}{(-az;q)_\infty}\ {}_2\phi_1\left(\begin{matrix} a,aq \\ qb^2 \end{matrix};q^2,z^2\right) = \frac{(az^2;q^2)_\infty}{(z,-az;q)_\infty}\ {}_2\phi_2\left(\begin{matrix} a,a^{-1}b^2 \\ qb^2,az^2 \end{matrix};q^2,az^2q\right). $$

Then the two equalities in (iii) are the limit case $a\to 0$ of the above two equalities. In the two equalities in (iii), with b replaced by q^b and z by $(1-q)z$, we obtain for $q\to 1$ that

$$ {}_1F_1\left(\begin{matrix} b \\ 2b \end{matrix};2z\right) = \mathrm{e}^z\ {}_0F_1\left(\begin{matrix} - \\ b+\frac12 \end{matrix};\tfrac14 z^2\right) = \mathrm{e}^z\ {}_0F_1\left(\begin{matrix} - \\ b+\frac12 \end{matrix};\tfrac14 z^2\right). $$

Equivalently, see Erdélyi [1953, Vol. 2, 7.2(3)],

$$ J_\nu(z) := \frac{(\frac12 z)^\nu}{\Gamma(\nu+1)}\ {}_0F_1\left(\begin{matrix} - \\ \nu+1 \end{matrix};-\tfrac14 z^2\right) = \frac{(\frac12 z)^\nu \mathrm{e}^{-\mathrm{i}z}}{\Gamma(\nu+1)}\ {}_1F_1\left(\begin{matrix} \nu+\frac12 \\ 2\nu+1 \end{matrix};2\mathrm{i}z\right). $$

On the q-level, with notation as in Exercise 1.24, the equalities in Exercise 3.2(iii) can be equivalently written as

$$ J_\nu^{(1)}(z;q^2) = \frac{1}{(-\frac14 z^2;q^2)_\infty}\ J_\nu^{(2)}(z;q^2) = \frac{(q^{2\nu+2};q^2)_\infty}{(q^2;q^2)_\infty}\ \frac{(\frac12 z)^\nu}{(-\frac12 \mathrm{i}z;q)_\infty}\ {}_2\phi_1\left(\begin{matrix} q^{\nu+\frac12},-q^{\nu+\frac12} \\ q^{2\nu+1} \end{matrix};q,\tfrac12 \mathrm{i}z\right). $$

The first equality in this last formula is also given in Exercise 33.2(iii), and it is attributed there to Hahn [1949c].

Note that by (i) respectively (iii) the functions $(az,-z;q)_\infty\ {}_3\phi_2\left(\begin{smallmatrix} a,b,-b \\ b^2,az \end{smallmatrix};q,-z\right)$ and $(z;q)_\infty\ {}_2\phi_1\left(\begin{smallmatrix} b,-b \\ b^2 \end{smallmatrix};q,z\right)$ are even in z.

By the expression given above for $J_\nu^{(1)}(z;q^2)$ the product formula

$$ {}_2\phi_1\left(\begin{matrix} a,-a \\ a^2 \end{matrix};q,z\right)\ {}_2\phi_1\left(\begin{matrix} b,-b \\ b^2 \end{matrix};q,-z\right) = {}_4\phi_3\left(\begin{matrix} ab,-ab,abq,-abq \\ a^2q,b^2q,a^2b^2 \end{matrix};q^2,z^2\right) $$

(see formula (4.9) in H. M. Srivastava & V. K. Jain, *q-Series identities and reducibility of basic double hypergeometric functions*, Canad. J. Math. 38 (1986), 215–231, and formula

(2.1) in M. J. Schlosser, *q-Analogues of two product formulas of hypergeometric functions by Bailey*, in *Frontiers in orthogonal polynomials and q-series*, World Scientific, 2018, pp. 445–449) can be rewritten as

$$J_\mu^{(1)}(z;q^2)J_\nu^{(1)}(z;q^2) = \frac{(q^{2\mu+2},q^{2\nu+2};q^2)_\infty}{(q^2,q^2;q^2)_\infty}\,\frac{(\frac12 z)^{\mu+\nu}}{(-\frac14 z^2;q^2)_\infty}\\ \times\ {}_4\phi_3\left(\begin{matrix} q^{\mu+\nu+1},-q^{\mu+\nu+1},q^{\mu+\nu+2},-q^{\mu+\nu+2} \\ q^{2\mu+2},q^{2\nu+2},q^{2\mu+2\nu+2}\end{matrix};q^2,-\tfrac14 z^2\right).$$

For the $q=1$ limits of these product formulas see formulas (16.12.1) and (10.8.3) in DLMF, `https://dlmf.nist.gov/`.

p. 147, Exercise 5.10:
In the numerator on the left-hand side replace e/ab and q^2f/e by c/qf and q^2f/c
(error observed in p. 841 of W. Groenevelt & E. Koelink,
J. Approx. Theory 163 (2011), 836–863).
The formula with the same error occurs in (7.2.6) in the book
L. J. Slater, *Generalized hypergeometric functions*, Cambridge University Press, 1966.
A reference for Exercise 5.10 with the correct formula is formula (5) in
L. J. Slater, *General transformations of bilateral series*,
Quart. J. Math., Oxford Ser. (2) 3 (1952), 73–80.

p. 152, Exercise 5.26: (communicated by Slobodan Damjanovic)
On line 2 replace the denominator parameter ad/d by aq/d.

p. 189, (7.5.7): On the second line the comma after $d\,e^{-i\theta}$ should be deleted.

p. 189, (7.5.8): Insert "q," after the second semicolon of the first ${}_8W_7$.

p. 212, l.5: (communicated by Slobodan Lj. Damjanovic)
In the ${}_5\phi_4$ insert a numerator parameter $\sqrt{q}$ and replace the denominator parameter $eq^{-2i\theta}$ by $qe^{-2i\theta}$.

p. 236, (8.8.19), l.4: (communicated by Slobodan Lj. Damjanovic)
In the ${}_6\phi_5$ replace the numerator parameter abz by az/b.
The same correction should be made in formula (4.11) of the paper

G. Gasper and M. Rahman, *A non-terminating q-Clausen formula and some related product formulas*, SIAM J. Math. Anal. 20 (1989), 1270–1282.

p. 324, (11.5.5): Note that $\lambda = qa^2/(bcd)$, just as in (11.5.1). Furthermore it is helpful to observe that in the application of (11.5.1) to the right-hand side of (11.5.1) we replace in (11.5.1) a, b, c, d, e, f, q^{-n}, λ respectively by $\lambda = qa^2/(bcd)$, $\lambda b/a$, f, $\lambda aq^{n+1}/(ef)$, e, $\lambda d/a$, q^{-n}, eq^{-n}/b. In the resulting identity apply (11.2.50) in order to obtain (11.5.5).

A version of (11.5.5) was given formula (3) in the paper
H. Rosengren, *New transformations for elliptic hypergeometric series on the root system A_n*, Ramanujan J. 12 (2006), 155–166.

I thank Slobodan Damjanovic for this reference. Formula (11.5.5) can be obtained from (3) by exchanging e and g, replacing N by n, and then applying (11.2.50) to the two elliptic shifted factorials in the quotient $(aq/(eg);q,p)_n/(aq/g;q,p)_n$.

p. 392, Jackson, F. H. (1905a): This paper appeared in 1904.

p. 403, Rahman, M. (1988b): (communicated by Slobodan Damjanovic)
Replace **33** (4), 111–120 by **31** (4), 467–476.

p. 420, list of Jackson, F. H.: Move the number 138 to the list of Jackson, M.