Errata and comments on the book Basic hypergeometric series, Second edition, by G. Gasper and M. Rahman

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p. 101, Exercise 3.2(iii):
We can combine the two equalities in (i) and (ii) as
\[ \phi_2\left(\frac{a,b,-b}{b^2,-az^2};q,z\right) = \phi_1\left(\frac{a,aq}{q^2};q^2,z^2\right) = \phi_2\left(\frac{a,a^{-1}b^2}{q^2,az^2};q^2,az^2q\right). \]
Then the two equalities in (iii) are the limit case \(a \to 0\) of the above two equalities. In the two equalities in (iii), with \(b\) replaced by \(q^{\frac{b}{2}}\) and \(z\) by \((1-q)z\), we obtain for \(q \to 1\) that
\[ \text{Equivalently, see Erdélyi [1953, Vol. 2, 7.2(3)],} \]
\[ J_{\nu}(1) = \left(\frac{1}{2}\right)^{\nu} \\ \Gamma(\nu+1) \left(\frac{\nu+1}{2}\right)^{-\nu} 2F_1\left(\frac{\nu+\frac{1}{2}}{2\nu+1};2iz\right) = \left(\frac{\nu+1}{2}\right)^{-\nu} 2F_1\left(\frac{\nu+\frac{1}{2}}{2\nu+1};2iz\right). \]

On the \(q\)-level, with notation as in Exercise 1.24, the equalities in Exercise 3.2(iii) can be equivalently written as
\[ J_{\nu}(1)(z;q^2) = \frac{1}{\left(\frac{1}{2}z^2;q^2\right)_\infty} J_{\nu}(2)(z;q^2) = \frac{(q^{2\nu+2};q^2)_\infty}{(q^2;q^2)_\infty} \frac{\left(\frac{1}{2}z\right)^\nu}{\Gamma(\nu+1)} 2F_1\left(\frac{\nu+\frac{1}{2}}{2\nu+1};q^{2\nu+1},q^2;2iz\right). \]
The first equality in this last formula is also given in Exercise 33.2(iii), and it is attributed there to Hahn [1949c]. Note that by (i) respectively (iii) the functions \((az,-z;q)_\infty 3\phi_2\left(\frac{a,b,-b}{b^2,az};q,-z\right)\) and \((z;q)_\infty 2\phi_1\left(\frac{b,-b}{b^2};q,z\right)\) are even in \(z\).

By the expression given above for \(J_{\nu}(1)(z;q^2)\) the product formula
\[ \phi_1\left(\frac{a,-a}{a^2};q,z\right) 2\phi_1\left(\frac{b,-b}{b^2};q,-z\right) = 4\phi_3\left(\frac{ab,-ab,abq,-abq}{a^2q,b^2q,a^2b^2};q^2,z^2\right) \]
(see formula (4.9) in H. M. Srivastava & V. K. Jain, \(q\)-Series identities and reducibility of basic double hypergeometric functions, Canad. J. Math. 38 (1986), 215–231, and formula
(2.1) in M. J. Schlosser, *q*–*Analogues of two product formulas of hypergeometric functions by Bailey*, in *Frontiers in orthogonal polynomials and *q*–series*, World Scientific, 2018, pp. 445–449) can be rewritten as

\[ J^{(1)}_{\mu}(z; q^2) J^{(1)}_{\nu}(z; q^2) = \frac{(q^2 z^2; q^2)_{\infty}}{(q^2; q^2)_{\infty}} \frac{(-q^{2 \mu+2}; q^2)_{\infty}}{(-1/4, q^2)_{\infty}} \times {}_4\phi_3 \left( \begin{array}{c} q^{\mu+\nu+1}, -q^{\mu+\nu+1}, q^{\mu+\nu+2}, -q^{\mu+\nu+2} \\ q^{2\mu+2}, q^{2\nu+2}, q^{2\mu+2\nu+2} \\ q^2, -\frac{1}{4} z^2 \end{array} \right). \]

For the \( q = 1 \) limits of these product formulas see formulas (16.12.1) and (10.8.3) in DLMF, https://dlmf.nist.gov/.

p. 147, Exercise 5.10:
In the numerator on the left-hand side replace \( e/ab \) and \( q^2 f/e \) by \( c/qf \) and \( q^2 f/c \) (error observed in p. 841 of W. Groenevelt & E. Koelink, J. Approx. Theory 163 (2011), 836–863).


p. 152, Exercise 5.26: (communicated by Slobodan Damjanovic)
On line 2 replace the denominator parameter \( ad/d \) by \( aq/d \).

p. 189, (7.5.7): On the second line the comma after \( d e^{-i\theta} \) should be deleted.

p. 189, (7.5.8): Insert “\( q \)” after the second semicolon of the first \( 8W_7 \).

p. 212, 1.5: (communicated by Slobodan Lj. Damjanovic)
In the \( s_5 \phi_4 \) insert a numerator parameter \( \sqrt{q} \) and replace the denominator parameter \( eq^{-2i\theta} \) by \( q e^{-2i\theta} \).

p. 236, (8.8.19), 1.4: (communicated by Slobodan Lj. Damjanovic)
In the \( s_6 \phi_5 \) replace the numerator parameter \( abz \) by \( az/b \). The same correction should be made in formula (4.11) of the paper G. Gasper and M. Rahman, *A non-terminating \( q \)-Clausen formula and some related product formulas*, SIAM J. Math. Anal. 20 (1989), 1270–1282.

p. 324, (11.5.5): Note that \( \lambda = qa^2/(bcd) \), just as in (11.5.1). Furthermore it is helpful to observe that in the application of (11.5.1) to the right-hand side of (11.5.1) we replace in (11.5.1) \( a, b, c, d, e, f, q^{-n}, \lambda \) respectively by \( \lambda = qa^2/(bcd) \), \( \lambda b/a, f, \lambda aq^{n+1}/(ef), e, \lambda d/a, q^{-n}, eq^{-n}/b \). In the resulting identity apply (11.2.50) in order to obtain (11.5.5).

A version of (11.5.5) was given formula (3) in the paper H. Rosengren, *New transformations for elliptic hypergeometric series on the root system \( A_n \)*, Ramanujan J. 12 (2006), 155–166.
I thank Slobodan Damjanovic for this reference. Formula (11.5.5) can be obtained from (3) by exchanging $e$ and $g$, replacing $N$ by $n$, and then applying (11.2.50) to the two elliptic shifted factorials in the quotient $(aq/(eg); q, p)_n/(a_q/g; q, p)_n$.

p. 392, Jackson, F. H. (1905a): This paper appeared in 1904.

Replace 33 (4), 111–120 by 31 (4), 467–476.

p. 420, list of Jackson, F. H.: Move the number 138 to the list of Jackson, M.