Errata to and remarks on books on Colombeau theory

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This concerns the two books

[O92] M. Oberguggenberger, *Multiplication of distributions and applications to partial differential equations*, Pitman Research Notes in Mathematics Series 259, Longman Scientific & Technical, 1992.

[GKOS] M. Grosser, M. Kunzinger, M. Oberguggenberger and R. Steinbauer, *Geometric theory of generalized functions with applications to general relativity*, Mathematics and its Applications 537, Kluwer Academic Publishers, 2001.

[GKOS, p.3, l.2]: Here a weakly L^2-convergent subsequence is meant, see [O92, p.24]. Such a subsequence exists because the closed unit ball in a Hilbert space is weakly compact and since the weak topology of the closed unit ball in a separable Hilbert space is metrizable (see [1, Problem 18]). Now use that compact metric spaces have the *Bolzano-Weierstrass property*, i.e., that every sequence has a convergent subsequence (see for instance [3, §24]).

[GKOS, p.59, Proof of Proposition 1.4.2]: By mere linear algebra the ϕ_β would only be in the algebraic dual of $\mathcal{D}'(\Omega)$, which is much larger than $\mathcal{D}(\Omega)$. A more precise argument, given in [O92, p.84] uses the Hahn-Banach theorem for extension to a continuous linear functional on $\mathcal{D}'(\Omega)$ of a linear functional initially defined on a finite subspace. Note that here the appropriate topology on $\mathcal{D}'(\Omega)$ as a dual of $\mathcal{D}(\Omega)$ is the strong dual topology. The strong dual of $\mathcal{D}'(\Omega)$ coincides with $\mathcal{D}(\Omega)$ since $\mathcal{D}(\Omega)$ is a Montel space. See [2, Chap. 3, Théor. XiV], [4, Prop. 34.4, Corollary to Prop. 36.9]. Here a *Montel space* is a locally convex Hausdorff space in which every absorbing, convex, balanced, closed subset is a neighbourhood of zero and every closed bounded subset is compact (see [4, Definitions 33.1 and 34.2]). By the way, this immediately implies that the only normable Montel spaces are the finite dimensional spaces. In particular, $\mathcal{D}(\Omega)$ is not normable.

References

[1] P. R. Halmos, A Hilbert space problem book, Van Nostrand, 1967.

[2] L. Schwartz, *Théorie des distributions*, Hermann, Paris, 1966.

[3] G. F. Simmons, *Introduction to topology and modern analysis*, McGraw-Hill, 1963.

[4] F. Treves, *Topological vector spaces, distributions and kernels*, Academic Press, 1967.