Errata, comments, additions to the book 'A GUIDE TO QUANTUM GROUPS' by Vyjayanthi Chari and Andrew Pressley, Cambridge University Press 1994, ISBN 0 521 43305 3, emerging from the study group at the Universiteit van Amsterdam, 1995. Collected by Erik Koelink (at present at TU Delft, see http://fa.its.tudelft.nl/~koelink/); edited and maintained by Tom Koornwinder (Univ. of Amsterdam). Version of August 26, 2005.

p. 19, \downarrow 2. Insert 'if ω is closed' after 'then'.

p. 19, \downarrow 3. Insert 'symplectic' before 'Poisson'.

p. 20, \downarrow 8. Add: 'By the way, the fact that { , } is a Poisson structure is already clear from (7).'

p. 21. Warning: This follows more or less immediately by taking $g' = g^{-1}$ in equation (8), page 22.

p. 23, \downarrow 3. Insert ' $(\pi'_g \otimes \pi'_g)$ ' in front of ' $(w_G)_g$ '.

p. 23, \uparrow 8. Note also that this bivector satisfies (8).

p. 23, \uparrow 7. Replace $(Ad_g \otimes Ad_g)(r) = 0$ by $(Ad_g \otimes Ad_g)(r) = r$.

p. 24, \downarrow 6. Replace $(1 - |a|^2 + |b|^2)$ by $(-1 + |a|^4 - |b|^4)$.

p. 28, \downarrow 12. In the displayed formula delete $\gamma_r^{aq} c_{sq}^p X_p$, i.e. the first term on the second line.

p. 30. In example 1.3.9 $\mathfrak{p}_{-} = u^{-1}\mathfrak{a}[[u^{-1}]]$, i.e. \mathfrak{p}_{-} is the space of formal power series in u^{-1} with coefficients in \mathfrak{a} with constant term equal to zero.

p. 31. In third displayed formula replace ' dx^k ' in the numerator by ' $d\theta^k$ '.

p. 49, \downarrow 3. Wilde & Lecomte (1988) doesn't occur in the References.

p. 52, \uparrow 4. Replace '[[X, a_i], a_j] \otimes $b_i \otimes b_j$ ' by '[[X, a_i], a_j] \otimes $b_j \otimes b_i$ '.

p. 53, \uparrow 10 Replace \mathfrak{g} by $\mathfrak{g} \otimes \mathfrak{g}$.

p. 67. Line 4 in the 2nd paragraph. Replace 'double G^* ' by 'dual G^* '.

p. 100, \uparrow 2. Replace 'qiantum' by 'quantum'.

p. 101. Definition 4.1.1, l.3: replace $A \to A$ by $k \to A$.

p. 102, Definition 4.1.2: The following symbolic notation (invented by Sweedler (1969), section 1.2) for comultiplication is often helpful: $\Delta(a) = \sum_{(a)} a_{(1)} \otimes a_{(2)}, (\Delta \otimes id) \Delta(a) = (id \otimes \Delta) \Delta(a) = \sum_{(a)} a_{(1)} \otimes a_{(2)} \otimes a_{(3)},$ etc.

p. 103. Last line. $S^B \circ \varphi = \varphi \circ S^A$ is a consequence of φ being an algebra and coalgebra morphism. This follows by an argument similar to the one on p. 104, Remarks [4]. For the details, see for instance M. Hazewinkel, 'Formal Groups', Prop. (37.1.10).

p. 104. Remark [3]: see Abe (1980), Theorem 2.1.4

p. 104. Remark [4]: note that $\iota \circ \epsilon$ is the unit element for the convolution product.

p. 105. *h*-Adic topology: see C. Kassel, 'Quantum Groups', §16.1 or N. Koblitz, '*p*-Adic Numbers, *p*-Adic Analysis, and Zeta-Functions', p. 88, ex. 25.

p. 106, Example 4.1.6. Some authors (for instance Abe (1980), p.170) define an affine algebraic group G by first defining its algebra $\mathcal{F}(G)$ of regular functions as a finitely generated commutative Hopf algebra A. Next they define G as the set of algebra homomorphisms $x: A \to k$. The Hopf algebra structure of A then yields a group structure on G by duality. Usually it is assumed here that the field k is algebraically complete. If moreover k has characteristic 0, then A has no nilpotent elements and G separates points on A.

p. 107, Example 4.1.8. Note also that the antipode S will be extended from \mathfrak{g} to $U(\mathfrak{g})$ as an antimultiplicative homomorphism.

p. 108, second paragraph. See for this result Milnor & Moore (1965), Proposition 7.21.

p. 109, second paragraph. The notions of left A-module algebra and left A-module coalgebra can be defined more conceptually after the tensor product of two left A-modules (p.110, last paragraph) and the trivial A-module (p.110, Example 4.1.12) have been defined. Then a left A-module algebra V can be characterized as an algebra and left A-module such that multiplication μ_V and unit ι_V are homomorphisms of left A-modules. Also, a left A-module coalgebra V can be characterized as a coalgebra and left A-module such that Δ_V and ϵ_V are homomorphisms of left A-modules.

p. 110, \downarrow 4. Insert $\circ\sigma$ after $(id_V \otimes S)$, where σ is the flip.

p. 110, last paragraph. Similarly to the definition of the tensor product of two representations of a bialgebra A, one can define the tensor product of two corepresentations ρ_1 and ρ_2 of A on V_1 and V_2 , respectively, as the corepresentation of A on $V_1 \otimes V_2$ given by $(id \otimes id \otimes \mu)(id \otimes \sigma \otimes id)(\rho_1 \otimes \rho_2)$. Together

with the notion of trivial corepresentation (Example 4.1.12) this makes it possible to give on p.109 a more conceptual definition of right A-comodule algebra and right A-comodule coalgebra.

p. 111, second paragraph, line 8. The vector space isomorphism $\operatorname{Hom}_k(V, W) \cong V^* \otimes W$ becomes an isomorphism of representations of A if we make $\operatorname{Hom}_k(V, W)$ into a representation of A by setting $a.f(v) = \sum_i a^i f(S(a_i).v)$ instead of the representation given in the book. However, in the case V = Wa good reason for choosing the representation on $\operatorname{End}_k(V)$ as given in the book, is that then the mapping $\lambda_V: A \to \operatorname{End}_k(V)$ intertwines the representation ad of A on A and the representation of A given in the book on $\operatorname{End}_k(V)$.

p. 112, \uparrow 1. Replace the exponent λ_1 of x_d by λ_d .

p. 113, \downarrow 12. For this equation \mathfrak{g} has to be abelian.

p. 113, last paragraph. See Sweedler (1969), Proposition 6.0.3.

p. 114, second paragraph: the term *I*-adic (*I* two-sided ideal of a ring *R*) occurs in p. 414 of the book 'Lessons on Rings, Modules and Multiplicities' by D.G. Northcott (1968) in connection with *I*-adic filtrations on *R*-modules.

p. 114, \downarrow 16. Replace 'an isomorphism' by 'injective'.

p. 114, Example 4.1.17, l.2: replace $\mathcal{F}(G)^{\circ} \cong U(\mathfrak{g})$ by $U(\mathfrak{g})$ is isomorphically embedded in $\mathcal{F}(G)^{\circ}$. See Abe (1980), p. 197 for the full characterisation of $U(\mathfrak{g})^{\circ}$.

p. 114, Example 4.1.17, l.13: replace $U(\mathfrak{g})^{\circ} \cong \mathcal{F}(G)$ by ' $\mathcal{F}(G)$ is isomorphically embedded in $U(\mathfrak{g})^{\circ}$ '. See for instance remarks in A. Joseph, 'Quantum Groups and their Primitive Ideals', §2.1.11.

p. 115. Part E: see Abe (1980), p. 154, §3.3.

p. 116. Example 4.1.19: See Hochschild, 'The Structure of Lie Groups', §II.3.

p. 121, \downarrow 7. Read 'Applying $\mu(\mu \otimes id)\sigma_{13}(id \otimes S \otimes S^2)$ ' instead of 'Applying $id \otimes S \otimes S^2$ '.

p. 122, $\downarrow 4$. In the expression for u_4 replace S by S^{-1} .

p. 122, \downarrow 9. Replace $S^{-1} \otimes S$ in the expression for u_1^{-1} by $S^{-1} \otimes S^{-1}$.

p. 122, last paragraph, lines 6–8. This argument is not convincing, since $\operatorname{End}_k(V)$, as a linear space, can also be naturally identified with $V^* \otimes V$, such that under this identification the trace functional on $V^* \otimes V$ becomes the usual trace functional on $\operatorname{End}_k(V)$. This identification will also give a left A-module structure to $\operatorname{End}_k(V)$ under which tr: $\operatorname{End}_k(V) \to k$ commutes with the action of A. However, a good reason for identifying $\operatorname{End}_k(V)$ with $V \otimes V^*$ is given by the above comment to p. 111, second paragraph, line 8.

p. 124, \downarrow 1. Replace '4.2.5' by '4.2.4 and from (16)'.

p. 124, \uparrow 9. Replace ϵ by $(\iota \circ \epsilon)$ and replace 1 by $1 \otimes 1$.

p. 126, Definition 4.2.9. One can use the definition of quantum trace in order to define the quantum character of the representation ρ : $\operatorname{qch}_{\rho}(b) := \operatorname{qtr}(\rho(b)) = \operatorname{tr}(\rho(gb))$. Then, if we write $\operatorname{ad}(a)a'$ for $\operatorname{ad}(a \otimes a')$ in Example 4.1.13, we have: $\operatorname{qch}_{\rho}(\operatorname{ad}(a)b) = \epsilon(a)\operatorname{qch}_{\rho}(b)$ and $\operatorname{qdim}(V) = \operatorname{qch}_{\rho}(1)$.

p. 127. Replace the last displayed formula by

$$\sum_{i,k} S^C(c_i)c_k \otimes b_k b_i = (S^C \otimes id)((S^{C^{-1}} \otimes id)(\mathcal{R}).\mathcal{R})$$
$$= (S^C \otimes id)(\mathcal{R}).\mathcal{R} = \mathcal{R}^{-1}\mathcal{R} = 1 \otimes 1.$$

p. 128, \uparrow 3. Replace A^* by $(A^*)^{op}$.

p. 129. Replace in the first displayed formula Δ by $(\Delta \otimes id)\Delta$ and $(S^{-1} \otimes id)$ by $(S^{-1} \otimes id \otimes id)$ and replace in the second displayed formula Δ^{op} by $(\Delta^{\text{op}} \otimes id)\Delta^{\text{op}}$.

p. 132. The second formula must start with $\langle \gamma^2, x \rangle$ instead of $\langle g^2, x \rangle$.

p. 132. In first displayed formula in (b) replace ' $\mu = \nu = 0$ ' by ' $\mu = \rho = 0$ '

p. 133, \uparrow 2. Replace 'van Daele' by 'Van Daele'.

p. 136, \uparrow 2. The notion of the zero object in the definition of additive category is missing.

p. 137. Condition (c) in the definition of an abelian category can be proved from the others. Take $f = m \circ e$ with m = ker(cokerf), e = coker(kerf) and this is unique up to isomorphism.

p. 138. Erratum to the comment above. Maclane (1971) has the extra condition ' $\rho_1 = \lambda_1$ ', but this can be proved from the other conditions. So the previous comment must be cancelled.

p. 139. In the lowest left diagram replace $id \uparrow and \uparrow \alpha$ by $id \downarrow and \downarrow \alpha^{-1}$.

p. 143, \uparrow 9. braid is not rigid, and on p. 144 figure 5 is false, since it is not a braid.

p. 144, \uparrow 1. It is not possible to define equivalence for tangles as for braids.

p. 150, def. 5.2.1. (ii) is a consequence of $\lambda_1 = \rho_1$, a condition which is not required in the definition for monoidal categories on p. 138 but can be proved from it; see a later comment for p. 138.

p. 153. Replace $\sigma_{W\otimes V,U}$ by $\sigma_{U,W\otimes V}$ in the diagram.

p. 174. In Definition-Proposition 6.1.2 replace '+' by '–'. d^\prime and $d^{\prime\prime}$ commute.

p. 188, \downarrow 5. Replace ' φ ' by ' φ_h ' twice.

p. 193, \downarrow 9. Replace ' $f_n H^n$ ' by ' $f_n H^n/n!$ '.

p. 195, last line. Replace '1.3.16', which doesn't exist, by '1.3.14'.

p. 216, \uparrow 10. Replace the expression for $\Delta(b)$ by $\Delta(b) = a \otimes b + b \otimes d$.

p. 229, last paragraph. $\mathcal{F}_R(M_m)$ is a well-defined algebra. Why is the QYBE a reflection of the associativity in $\mathcal{F}_R(M_m)$?

p. 232, \downarrow 13. Replace in the Ore condition 'ms' = m's' by 'ms' = sm''.

p. 232, \downarrow 15. See §2.1 of McConnell & Robson (1987). (In the references 'McConnel' is before 'Ma...'. See also A.V. Jategaonkar, 'Left Principal Ideal Rings', LNM 123, Springer, 1970, §1 for the construction of the localisation in non-commutative rings using an equivalence relation. It is not sufficient that S is multiplicatively closed and satisfies the Ore condition in order to construct the localisation Q of \mathcal{M} at S. To formulate the other condition define $\mathcal{A} = \{m \in \mathcal{M} : \exists s \in S, ms = 0\}$, which is an ideal in \mathcal{M} by the Ore condition. Let $\overline{\mathcal{M}} = \mathcal{M}/\mathcal{A}$ be the quotient ring, then $\overline{S} = S/\mathcal{A} \subset \overline{\mathcal{M}}$ has to consist of regular elements, i.e. xs = 0 implies x = 0 and sx = 0 implies x = 0 for $x \in \overline{\mathcal{M}}, s \in \overline{S}$. Moreover, \mathcal{M} can be injectively embedded in Q if and only if $\mathcal{A} = \{0\}$ or equivalently, all elements of S are left regular.

p. 232, last paragraph. If we also want the natural injection $\mathcal{F}_R(M_m) \hookrightarrow \mathcal{F}_{R,q}(GL_m)$, then we also need that *qdet* is not a zero-divisor in $\mathcal{F}_R(M_m)$.

p. 235, \downarrow 15, 18. Replace $\langle a_0 \otimes b_0, \delta(X_0) \rangle$ by $\langle (da_0)_e \otimes (db_0)_e, \delta(X_0) \rangle$.

p. 237, \uparrow 4. Replace \odot by a boxed dot.

p. 238. The formula on the last line must have formula number (21).

p. 239, Remark [2]. In the Dipper and Donkin case, cf. remark [1], this doesn't correspond to Dipper and Donkin (1991), (4.1.9).

p. 239. In the last displayed formula add the subscript i to each of the H, X^{\pm} that doesn't have a subscript.

p. 254, \uparrow 9. The Hochschild complex is treated in Ch. 6, not in Ch. 4.

p. 258. In remark [5] put $\omega(h) = h$, $\phi(H_i) = -H_i$, $\phi(h) = h$. This gives an anti-automorphism ω and automorphism ϕ of $U_h(\mathfrak{g})$, cf. Definition-Proposition 6.5.1. (Check (34).)

p. 259, \downarrow 14, p. 392, before thm. 12.2.1, p. 394, \downarrow 14. Beck (1993) doesn't occur in the References.

p. 264, (9). Replace q_i by q in the q-factorial in the denominator.

p. 267. In Proof of Proposition 8.3.3 need to consider the generic element $a = \sum_{s+t>0} f_{s,t}(h)H^s(X^+)^t$. It's clear that hH and hX^+ are in $U_h^{\geq 0}$. Consider an element $a = \sum_{s+t>0} f_{s,t}(h)H^s(X^+)^t \in U_h^{\geq 0}$, where $f_{s,t}(h) \in \mathbb{C}[[h]]$. Choose any σ, τ such that $\sigma + \tau > 0$ and consider $\Delta^{(\sigma+\tau)}(a)$. We look at its expression with respect to the obvious basis for the tensor product. The coefficient of the element of the basis $H^{\otimes \sigma} \otimes (X^+)^{\otimes \tau}$ is given by $\sigma![\tau]_{e^h}!\tau^{-1/2\tau(\tau-1)}f_{\sigma\tau}(h) \cong \sigma!\tau!f_{\sigma\tau}(h)(mod\ h^k), \forall k \ge 1$. The proof of this statement is a a consequence of the proof of lemma 8.3.4, first formula on p. 269, and a counting argument. Therefore, $\Delta_{\sigma+\tau}(a) \equiv 0 \mod h^{\sigma+\tau}$ implies $h^{\sigma+\tau}|f_{\sigma,\tau}(h)$.

p. 268, \downarrow 1. Replace $f_{s+t}(h)$ by $f_{s,t}(h)$ and t! by $q^{-t(t-1)/2}[t]_q!$.

p. 268, \downarrow 7. Replace 'let ξ and η ' by 'let η and ξ '. Thus $\langle \xi, \tilde{H} \rangle = 1$ and $\langle \eta, \tilde{X}^+ \rangle = 1$ and zero on other monomials.

p. 270, \uparrow 7. Insert $S^{-1} \otimes id$ in front of $\Delta_h^{(2)}(X^+)$.

p. 270, \uparrow 11. Replace in the right hand side of the formula for $\alpha.a$ all indices j for any a, a', a'' by the index i.

p. 270, \uparrow 9,13. Replace $(S_h^{-1} \otimes id)$ by $(S_h^{-1} \otimes id \otimes id).$

p. 271, \uparrow 8. Replace 8.2.7 by 8.2.6.

p. 281. Delete first black box.

p. 282, \uparrow 15, 16. Replace $e^{-d_j a_{ji} h H_j}$ and $e^{-d_i a_{ij} h H_j}$ by $e^{-d_j a_{ji} h H_i}$ and $e^{-d_i a_{ij} h H_i}$.

p. 286, \uparrow 6. Replace Flag $(V)_n$ by Flag $_n(V)$.

p. 293. Replace $q = \epsilon_i$ by $q = \epsilon$ in Proposition 9.2.13.

p. 295. In the displayed formula of example 9.2.17 interchange x^+ and k in the left hand side.

p. 298. In Proposition 9.3.3 replace U_q^{res+} , U_q^{res-} and U_q^{res0} by U_q^+ , U_q^- and U_q^0 .

p. 308, \downarrow 10. Replace $\mathcal{R}(\Gamma)$ by Rep (Γ) .

p. 308, \uparrow 11. Replace $e_{ij}e_i = me_je_{ij}$ by $e_{ij}e_j = me_je_{ij}$.

p. 319, \uparrow 17. Replace $V(\lambda)$ by $V_q(\lambda)$.

p. 330. Is Example 10.1.23 in correspondence with Vaksman and Korogodskiĭ (1991) and Masuda, Mimachi et al. (1990a,b). The strange series representations seem missing. Maybe this is a different quantum $\mathfrak{su}(1,1)$ algebra.

p. 350, \uparrow 6. Replace $\mathcal{V}_1 \times \mathcal{V}_2$ by $\mathcal{V} \times \mathcal{V}$.

p. 378, formula (13). Replace $H_{i,r+s}$ by $H_{i,r+s}/d_i$.

p. 381, formula (22). Replace $X_{i,r+1}^{\pm}$ by $d_i X_{i,r+1}^{\pm}$.

p. 381, formula (22). Replace $[X_{i,r+1}^+, X_{i,0}^-]$ by $d_i[X_{i,r+1}^+, X_{i,0}^-]$.

p. 381, proof prop. 12.1.6. Here the Poincaré-Birkhoff-Witt theorem is needed, cf. proposition 12.1.8.

p. 381, proof cor. 12.1.7. Centre of U(g[u]) equals centre of $U(g) \otimes C[u]$.

p. 387, \downarrow 9. Replace ev_a by ev^a twice in the displayed formula.

p. 392, \downarrow 10. Replace 6.1.5 by 6.5.1.

p. 392. In the fourth line of section \mathbf{A} replace 6.1.5 by 6.5.1.

p. 392, thm. 12.2.1. Definition 9.1.1 gives only $U_q(\mathfrak{g})$ for finite dimensional Lie algebra \mathfrak{g} . Probably use 6.5.1 and similar substitions.

p. 393, \uparrow 15. Replace A_q by $U_q(\tilde{\mathfrak{g}})$.

p. 405, \downarrow 1. Replace 'associative' by 'associative'.

p. 410, \uparrow 12. Replace 12.3.8 by 12.3.10.

p. 410, thm. 12.3.13. Replace 9.1.1 by 12.2.1.

p. 411, \uparrow 8. Replace 12.3.8 by 12.3.10.

p. 414, \downarrow 9. Replace all indices 'i' in the sum on the right hand side by indices 'j'.

p. 458. In Example 13.3.11, \uparrow 4. Interchange 'i' and 'e' in 'wieghts'.

p. 460. In formula (19) a minus sign should be inserted in the definition of $S(X^{-}) = -KX^{-}$.

p. 461. Proposition 13.4.2 and (24) are not correct. In particular, $\langle X^+, b \rangle = q^{-1/2}$ and $\langle X^-, c \rangle = q^{-1/2}$.

p. 463, top line. Replace the dot in the right hand side superscript ' λ . – μ ' by a comma.

p. 465. The paragraph following (25) is not entirely correct. Before the limit transition $q \uparrow 1$ you should replace a_i , b_i and z by q^{a_i} , q^{b_i} and $(1-q)^{1+s-r}z$ to see that the $_r\varphi_s$ -series tends to a hypergeometric $_rF_s$ -series.

p. 471. Replace 'Hansel-Lommel' by 'Hansen-Lommel'.

p. 471. Corollary 13.5.7 is not a consequence of the Schur orthogonality relations, but of the fact that the corepresentations are unitary, $\sum_{i} \rho_{k,i} \rho_{ji}^* = \delta_{kj}$. The *q*-Hankel orthogonality relations on p. 472 are the consequence of the Schur orthogonality relations up to a scalar.

p. 473. The analytic proof of the addition formula in Theorem 13.5.8 is not in the paper by Koornwinder and Swarttouw. An analytic proof, as well as a limit transition to Graf's addition formula as q tends to 1, is given in H.T. Koelink and R.F. Swarttouw, 'A q-analogue of Graf's addition formula for the Hahn-Exton q-Bessel function', J. Approx. Theory **81** (1995), 260–273. There is also a different proof of Theorem 13.5.8 in E.G. Kalnins, W. Miller and S. Mukherjee, 'Models of q-algebra representations: the group of plane motions', SIAM J. Math. Anal. **25** (1994), 513-527.

p. 482, \downarrow 11. Replace 'a said' by 'a is said'.

p. 495, \uparrow 7. Replace 'show' by 'shown'.

p. 507. Replace the first ρ_m in the first displayed formula by ρ_{m+1} .

p. 508. Insert '(' after the first ' $(-1)^{-m+1}$ ' in the first unnumbered displayed formula.

p. 515. In the second line of the first displayed formula replace $trace(trace_{m+1}(L(-2\pi)^{\otimes m}\beta(\ldots)))$ $trace(trace_{m+1}(L(-2\pi)^{\otimes m}\beta\otimes id(\ldots)))$

p. 515, prop. 15.2.11. The definition for $R_{-}(0)$ must be replaced by $\sigma(R_{+}(0))^{-1}\sigma$ or $(T \otimes T)(\mathcal{R}_{h}^{21})$.

p. 517. Definition 15.3.1 should exclude the case $C = \{0\}$.

p. 520, last picture. Change the above overcrossing into an undercrossing.

p. 523, $\downarrow 10$. Change $\tilde{F}(K, \lambda_K) = 1$ to $c^{-1} \sum_{\lambda_K \in \mathcal{C}} d_{\lambda_K} \tilde{\mathcal{F}}(K, \lambda_K) = 1$.

p. 523. In the last two displayed formules the factor $c^{-\sigma_{\leq 0}(L)}$ is missing. It is unclear how this affects the proof of the invariance under the Kirby +1 moves.

p. 562, \uparrow 4. Insert '±1' in front of ' $a_{ij}X_j^{\pm}$ '.

p. 565. Second displayed formula in 'A 6 Root Vectors'. The second $T_i(X_j^+)$ must be replaced by $T_i(X_j^-)$.

p. 574. Replace page numbers for the reference Birman and Wenzl (1989) by 249–273.

p. 619. Podleś (1991) is Podleś (1992c).

p. 632. Van Daele and Van Keer has appeared in Compositio Math. 91 (1994), 201-221.

p. 638, \downarrow 9. Replace A^0 by A° .

p. 645. The entry 'HOMFLY polynomial' must not be indented.

p. 648. Replace 'Hansel-Lommel' by 'Hansen-Lommel' in the entry for q-Bessel function.