Review of "Hypergeometric Summation" by Wolfram Koepf
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Wolfram Koepf, *Hypergeometric Summation. An Algorithmic Approach to Summation and Special Function Identities*,
Advanced Lectures in Mathematics, Vieweg, Braunschweig/Wiesbaden, 1998.
ISBN 3-528-06950-3, x+230 pp., 69.00 DM.
Free accompanying software can be downloaded from
`http://www.vieweg.de/welcome/downloads/supplements.htm`
or from `http://www.imn.htwk-leipzig.de/~koepf/research.html`

The book under review deals with the very successful set of algorithms for hypergeometric summation which came to maturity during the last decade. This development started with Gosper's algorithm for indefinite summation in 1978, and got a decisive impetus by Zeilberger's algorithm for definite summation in 1990. Closely related is the WZ-method, while Petkovšek's algorithm provides a useful complement. These algorithms were extended to the q-case and (in some cases) to multiple summation. Also, analogous algorithms were developed for the case where one or both variables become continuous and derivatives and integrals replace differences and summations. Implementations of the algorithms in many different computer algebra systems were given, in particular in Maple and Mathematica. The success of these implementations has been enormous. It is a serious option nowadays to replace compendia of formulas for hypergeometric functions, or their electronic versions containing static formulas, by electronic utilities where the formula of desired type is produced by an implemented algorithm. A further, very attractive property of the Gosper-Zeilberger algorithm is that it not just produces an explicit sum evaluation or recurrence (if it exists), but also provides a few simple data (proof certificate) which give all ingredients for a short proof of the identity. These developments might revolutionize the way mathematics will be done in future, as is frequently argumented by Doron Zeilberger in an eloquent but provoking way.

An account of these algorithms addressed to a wide audience was published in 1996 by Petkovšek, Wilf and Zeilberger in the book *A=B*. The book under review is very much related to *A=B* in scope and aimed audience, but in many respects it is also different. Let me first discuss the similarities. Both books can be read with few preliminaries. They do not require earlier acquaintance with special functions or computer algebra algorithms or with computer algebra systems in practice. In both books the "five basic algorithms" by Sister Celine, Gosper, Zeilberger, Wilf-Zeilberger and Petkovšek take a prominent place (Koepf treats W-Z before Zeilberger's algorithm). Each of the two books gives many worked out sessions with Maple and Mathematica (in Koepf's book only Maple). Both books effectively bring the reader from scratch to a good understanding and knowledge of these algorithms and to a practical ability to use them. However, none of the books gives fully rigorous proofs that the algorithms are valid. Both books point to websites from where implementations by the author(s) of the algorithms can be downloaded.

As for the differences, the style in *A=B* is looser, and the message that this topic is great fun is effectively sent to the reader. Standard hypergeometric notation is introduced in both books, and regularly used by Koepf, while *A=B* gives most sums with terms in the form of products of binomial coefficients. Koepf pays more attention to subtle aspects of the algorithms (for instance zeros occurring in the denominator), but he usually does not discuss these subtleties in an exhaustive way (see for instance the discussion about zeros in the denominator on various places in Ch. 6 dealing with the Wilf-Zeilberger method). Thus, if one wants to be definitely convinced that the algorithm is correct, one has to go to Koepf's implementation, and check the details of his Maple code. Koepf gives more extensions of the algorithms than A=B. Almost each chapter ends with a short discussion of the q-analogue. He discusses the extended Gosper algorithm and WZ method (for instance where in the evaluation of a definite sum different parities of n yield different analytic expressions). Very valuable additions, compared to A=B, are Koepf's chapters 10–13 on, respectively, Differential equations for sums, Hyperexponential antiderivatives, Holonomic equations for integrals, Rodrigues formulas and generating functions. Koepf has a wealth of exercises at the end of each chapter, many more than in A=B. Koepf's accompanying Maple source file `hsum.mpl` bundles all relevant procedures for single summation and single integration ($q = 1$), while his file `qsum.mpl` gives many procedures for the q-case. These files are somewhat more comprehensive than Doron Zeilberger's Maple source files `EKHAD` and `qEKHAD`. A very nice service of Koepf is that Maple worksheets for the various Maple sessions described in his book are available from his website.

So, as a conclusion the book under review, together with the accompanying free software, can be very much recommended for self-study, for reference, and for usage in classroom and student seminars.