Mittag-Leffler, the man, the mathematician and his network

Tom Koornwinder

Korteweg-de Vries Institute, University of Amsterdam
T.H.Koornwinder@uva.nl

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Gösta Mittag-Leffler (1846–1927)
Acta Mathematica

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Description
Publishes original research papers of the highest quality in all fields of mathematics. Acta Mathematica is a well established and highly respected international mathematics journal. It is published by the Institut Mittag-Leffler, a research institute of the Royal Swedish Academy of Sciences. Two volumes consisting of two issues each are published annually.

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Tom Koornwinder

Mittag-Leffler, the man, the mathematician and his network
1846  born March 16 in Stockholm
1855–61 at elementary school in Stockholm
1861–65 at Gymnasium in Stockholm
1865–72 student in Uppsala up to doctorate in mathematics
1872–73 and 1876–77 docent in Uppsala
1873–76 “postdoc” in Paris, Göttingen and Berlin
1877–81 professor in Helsingfors (Helsinki)
1881–1912 professor at Stockholm College
1881 first volume of Acta Mathematica
1882 marriage with Signe Lindfors (from Helsingfors)
1884 Sonya Kovalevsky becomes docent in Stockholm, in 1889 she becomes professor there.
1891  moves to new villa in Djursholm; death of Sonya K.
1892  death of his sister Anne Charlotte
1903  death of his mother
1906  final remodeling of his villa with big library room
1909  buys summer house in Tällberg (Dalarna)
1912  retires from Stockholm College
1916  publishes his will, establishes M-L Math Foundation
1921  death of his wife Signe
1927  dies on July 7
Father and mother

Johan Olof Leffler (1813–1884)    Gustava Mittag (1817–1903)

Both had paternal ancestors originating from Germany. Johan Olof studied humanities in Uppsala and became teacher and headmaster in Stockholm. Gösta’s last name Mittag-Leffler already taken at school.
Brothers and sister

left: Artur (b. 1854)
middle: Gösta (b. 1846)
right: Frits (b. 1847)
below: Anne Charlotte (b. 1849)

Frits Läffler became professor of Swedish language in Uppsala, but soon took permanent leave because of health problems.
Anne Charlotte became novelist and playwright.
Artur became engineer.
Tom Koornwinder  Mittag-Leffler, the man, the mathematician and his network
student years, 1865–1872

First by boat from Stockholm to Uppsala, later by train. Studies mathematics, physics (prof. Ångström) and astronomy. Some Latin is required, he does not like it. Very active in the students math association.

Earns some money by tutoring. Notably he was tutor of the son of finance minister Gustaf af Ugglas, and later of the brothers of the king’s royal stablemaster Othan Demirgian (Habbib Bey). Much involved in students’ and teachers’ opposition against new university legislation.

His thesis is not very remarkable in hindsight. Still he succeeds next to get the very prestigious Byzantine grant for a three years study abroad.

While visiting the conference for Scandinavian natural scientists, Copenhagen, 1873, he becomes aware of the poor quality of mathematics research and education in Uppsala.
postdoc in Paris and Berlin

Charles Hermite (1822–1901)
lectures on elliptic functions

Karl Weierstrass (1815–1897)
lectures on elliptic and abelian functions
Elliptic functions

Jacobi:

\[ x = \sin \phi, \quad u = \int_0^\phi \frac{ds}{\sqrt{1 - k^2 \sin^2 s}} = \int_0^x \frac{dt}{\sqrt{(1 - t^2)(1 - k^2 t^2)}}. \]

inverted as \( \phi = \text{am}(u, k) \). Then \( x = \text{sn}(u, k) := \sin(\text{am}(u, k)) \).

Also \( \sqrt{1 - x^2} = \text{cn}(u, k) := \sqrt{1 - \text{sn}^2(u, k)} \)

and \( \sqrt{1 - k^2 x^2} = \text{dn}(u, k) := \sqrt{1 - k^2 \text{sn}^2(u, k)} \).

Weierstrass:

\[ z = \int_\infty^w \frac{dt}{\sqrt{4t^3 - g_2 t - g_3}}, \]

inverted as \( w = \wp(z; g_2, g_3) \).

\( \text{sn}, \text{cn}, \text{dn} \) and \( \wp \) are elliptic functions, i.e., meromorphic doubly periodic functions.

They can be expressed in terms of quotients of theta functions, which have nice explicit expressions as Fourier series and as infinite products.
Abelian or hyperelliptic functions

Riemann surface $y^2 = p_m(x)$ with $p_m$ polynomial of degree $m$ has genus $g = \left\lfloor \frac{1}{2}(m - 1) \right\rfloor$.

Abelian integrals: $z = \int_a^w \frac{t^{j-1}}{\sqrt{p_m(t)}} \, dt \quad (j = 1, \ldots, g)$.

Abelian functions: $w_i(u_1, \ldots, u_g) \quad (i = 1, \ldots, g)$ obtained by inversion of the system

$$u_j = \sum_{i=1}^{g} \int_{a_i}^{w_i} \frac{t^{j-1}}{\sqrt{p_m(t)}} \, dt \quad (j = 1, \ldots, g).$$

These are $2g$-periodic meromorphic functions on $\mathbb{C}^g$ and can be written as quotients of (Riemann) theta functions in $g$ variables.

Schottky problem ($g > 1$): Period matrices depend on $\frac{1}{2}g(g + 1)$ parameters, while number of complex conformal moduli of Riemann surface is $3(g - 1)$. The difference $\frac{1}{2}(g - 2)(g - 3) > 0$ if $g > 3$. 
Completely analytic. Rare geometry only for illustration. According to ML this was a definite advantage: the Riemann approach failed in discovering properties of higher-order transcendents. ML: “Analysis is infinitely more general than geometry.”

No general definitions. No general theorems about functions. A function is a power series and all is deduced from the power series. (Not optimal in ML’s view.)

Utter clarity and precision of proofs. No metaphysics.

Bad formal presentation. “In France Weierstrass would not have been admitted as a university teacher.” Probably, therefore few of his students understood him completely.
ML declined offer by Weierstrass for possible position in Berlin.

Opening in Helsingfors because Lorenz Lindelöf (father of Ernst Lindelöf) became high public servant. He chaired the evaluation of candidates. Applicants had to write a treatise and to present this in a public lecture. ML wrote a treatise about elliptic functions. Hermite, Schering, Kronecker and Weierstrass sent letters of recommendation.

ML travelled in February from Berlin to Helsingfors via St. Petersburg. By accident the yet incomplete draft of his treatise arrived in Stockholm instead of Helsingfors. No chance to pick it up in winter by crossing the Baltic Sea. Only six weeks were left for writing it again from scratch and having it printed.

ML's presentation was very successful. However, much opposition against his candidacy by the movement for Finnish language. The final decision had to be taken in St. Petersburg. The letter of appointment came after one year.
Some of Mittag-Leffler’s mathematical results

Theorem (Mittag-Leffler, 1876)

For any sequence \( \{a_n\} \) of distinct points in \( \mathbb{C} \) tending to infinity and for any sequence of non-zero polynomials \( \{g_n\} \) vanishing at 0 there exists an analytic function having poles in \( a_n \) with Laurent tails \( g_n(1/(z - a_n)) \) \( (n = 1, 2, \ldots) \).

He extended this in 1884 to the case of an isolated sequence \( \{a_n\} \) in a domain \( U \) of the extended complex plane. So the sequence of poles may have as a limit point an essential singularity of the function to be found.

Mittag-Leffler function (1903): \( E_{\alpha,\beta}(z) := \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)} \). For \( \alpha > 0 \) an entire function of order \( 1/\alpha \).

Mittag-Leffler star (1899–1905): For an analytic function \( f \) on the unit disc let \( z \in S_f \) iff \( f \) has an analytic continuation along the ray from 0 to \( z \). There is a series of polynomials which converges to \( f \) on \( S_f \).
Signe Lindfors was 15 years younger than Gösta. Engagement in 1881, marriage in 1882. She was the daughter of wealthy and aristocratic parents in Helsingfors.

Sonya was hired at Stockholm College in 1884.

Anne Charlotte had a “mariage blanc” with Gustaf Edgren starting 1872. In 1888 she met Pasquale del Pezzo in Naples. They fell in love, married and got a son. Soon after she died.
Position at Stockholm College (1881–1912)

- Stockholm College started its educational activities in 1878. Until 1904 it was not allowed to grant degrees.
- ML fiercely opposed that it would become degree granting: If there was no focus on getting degrees, the school could attract the best students by offering advanced lectures.
- Was rector several times, never reelected after one year.
- ML had some very good students: Bendixson, Phragmén, von Koch, Fredholm.
- He had weekly seminars in the evening at his home.
- He invited famous guest lecturers to Stockholm: Painlevé and Volterra.
- For a while he was unhappy in Stockholm and made sondations for getting a position in the USA.
- In later years he often took leave in early spring for health reasons and went to Italy or a Swiss spa, doing under way a lot of business in Paris and Berlin.
Mittag-Leffler, the man, the mathematician and his network
C'est à la munificence de Sa Majesté le roi OSCAR II que nous devons d'avoir pu fonder le journal dont nous offrons la première livraison aux amis des mathématiques. L'Association en mémoire de Lars Hierta, la Fondation de Letterstedt ainsi que les personnes dont les noms suivent ici — C. J. Malmsten, Ch. Hermite, Fr. P. von Beijer, F. Kempe, H. R. Astrup, C. Ekeham, N. G. Sörensen, O. Wijk, Fr. Piper, O. Dickson, B. Kempe, W. Kempe, S. Axell, L. E. Rubensson, C. O. Rubensson — ont également contribué aux frais de l'œuvre dont l'auguste souverain a daigné se constituer le protecteur.


Der Zeitpunkt, zu welchem wir die Herausgabe beginnen, ist gewiss einer der fruchtbaren in der Geschichte der Mathematik, wegen der grossen Anzahl und Wichtigkeit der Entdeckungen auf dem Gebiete der Analysis. Dieses rege Leben ist durch die in verschiedenen

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*1882-87: Austria, Belgium, Holland, Switzerland, United States of America
1887-97: Austria, Belgium, England, Holland
Dutch contributions, 1882–1912
6 (1885) and three more:

12 (1889):

ÜBER GEWISSE EBENE CONFIGURATIONEN
VON
J. DE VRIES
in KAMPEN (Holland).

21 (1897):

A SPECIAL CASE OF DIRICHLET'S PROBLEM FOR TWO DIMENSIONS
BY
J. C. KLUYVER
of LEYDEN.

27 (1903):

SUR L'INTÉGRATION DES DIFFÉRENTIELLES BINÔMES
PAR
W. KAPTEYN
à UTRECHT.
Acta Mathematica

- truly international journal
- good balance between German and French contributions
- exclusively Scandinavian editorial board; ML dominating. Later Sonya K. important board member.
- also important work by editorial assistants: Eneström, Phragmén, Marcel Riesz (index volume 1–35 in 1913).
- financed by donations (King Oscar II, etc.), support by the Scandinavian countries, subscriptions from many countries; annual deficit paid by ML.
- every year foreign trips by ML to raise money and attract submissions.
- enemies of the journal (and ML): Kronecker and Schwarz.
- André Weil: *The Acta Mathematica were the product of his genius, while nothing more than talent went into his mathematical contributions.*
Henri Poincaré (1854–1915)

- already at young age seen by ML as a genius (his work on automorphic functions)
- exchanged 259 letters with ML
- informal style of writing
- 25 papers in Acta, notably the ones on rotating fluids and on divergent series

King Oscar II prize competition

- announced in 1885, deadline 1888, prize award in 1889
- choice of 4 topics, including the $n$-body problem
- jury of Hermite, Weierstrass and ML (chair); winning paper to be published in Acta
- Poincaré’s submission on $n = 3$ case got the prize
- many clarifications needed, by which the paper grew
- serious error discovered by Phragmén after printing; correction took one year; birth of the idea of chaos
Sonya Kovalevsky (1850–1891)

- Student of Weierstrass. Her thesis was about (i) Abelian integrals, (ii) the rings of Saturn, and (iii):

- **Cauchy-Kovalevsky theorem**
  \[ u_t = \sum_{j=1}^{n} a_j(t, x_1, \ldots, x_n, u) u_{x_j}, \text{ pde} \]
  for vector-valued \( u(t, x) \) with matrix-valued \( a_1, \ldots, a_n \) analytic near \( (0, \ldots, 0) \), has for \( u(0, x) = 0 \) unique local analytic solution \( u(t, x) \).

- \( u_{t} = u_{xx}, \quad u(0, x) = (1 - x)^{-1}, \)

- \[ u(t, x) = \sum_{k=0}^{\infty} \frac{(2k)!}{k!} \frac{t^k}{(1 - x)^{2k+1}} \] (not convergent)

- **Kovalevsky top**: general integrable case of rotation of solid body, using hyperelliptic functions in two variables. Awarded by Prix Bordin (1888).
Sonya wrote novels *Russian childhood* and *Nihilist girl*. She wrote plays together with Anne Charlotte. From an initially white marriage with Vladimir Kovalevsky a daughter is later born. During her last years a troublesome love affair with distant cousin Maxim Kovalevsky. *Too much happiness* about Sonya is the title story in Alice Munro's book.
Djursholm, new town for well-to-do middle class. ML bought largest lot; he had his own train stop.
Mittag-Leffler’s villa in Djursholm (cntd.)

View on the archipelago from the library room

The “gula hus” of Frits Läffler

“Number (speech) is the beginning and the end of thought. With thought, number (speech) is born. Without number (speech), thought goes nowhere.”

In 1916 there were about 40,000 volumes in the library.
Mittag-Leffler in business

- Starting 1882 for several decades board member and actuary of the Victoria Life Insurance Company.
- From about 1890 increasing involvement in business transactions, investments in new companies using new technologies, in the north of Sweden, Norway and Finland.
- For a while he owned a carbide (CaC$_2$) factory in Alby. $\text{CaO} + 3 \text{C} \longrightarrow \text{CaC}_2 + \text{CO}$ (at 2000 °C in an electric arc furnace, invented 1888). Next acetylene (C$_2$H$_2$) by $\text{CaC}_2 + 2 \text{H}_2\text{O} \longrightarrow \text{C}_2\text{H}_2 + \text{Ca(OH)}_2$.
- Sometimes desperate that no time was left for math.
- All efforts aimed at collecting enough money for establishing an Institute Mittag-Leffler.
Mittag-Leffler in politics

- Social feelings (insurance company, influence by Anne Charlotte andSonya) later diminished. He became a conservative and opposed the liberal party.
- Involved with reform of Swedish voting system
- Supported the case for Finnish independence from Russia.
- Regretted the dissolution of the Swedish-Norwegian union in 1905. To maintain Scandinavian ties he organized in 1909 the first Scandinavian math congress in Stockholm.
- In 1914 he supported the farmers who demonstrated at the royal palace against the soft Swedish attitude in strengthening the army. He accused the liberal prime minister to give Russian spies access to Sweden.
- Had sympathy for the Germans during World War I. After the war he opposed the boycott of German scientists in international organisations.
- Supported after World War I the independence movement in Åland.
- Could participate in the final votes as a member of the Academy of Science.
- Started sometimes intensive lobbies for a candidate for the physics prize.
- He succeeded with Marie Curie (twice) and Lorentz, but lost the case for Poincaré. He was sceptic about Einstein and Planck.
- Every year in December, after the official Nobel festivities, a grand party for the laureates and many others in his villa in Djursholm.
He studied mathematics in Gent, Belgium. While on holidays in Sweden in summer 1914 he stayed there during the war and became librarian of Mittag-Leffler. He had a very cordial relationship with ML. He married ML’s secretary Gunhild Sahlén. He returned to Belgium, but came again to Sweden in the thirties. He published philosophical works.
G. & S. MITTAG-LEFFLER
TESTAMENT 15/3 1916.

Auszug aus dem Testament, errichtet und unterschrieben von
G. MITTAG-LEFFLER und SIGNE MITTAG-LEFFLER, geb. AF LINDFORS

Hiermit erklären wir, unter Änderung des zwischen uns am 6. Januar 1883 errichteten Testamentes, dass unser letzter Wille ist, dass nach unserer beider Tod all unser zurückgelassenes Eigentum einer Stiftung zufällt, die den Namen tragen soll:

Mathematische Stiftung der Ehegatten Mittag-Leffler.

Die Aufgabe der Stiftung soll sein, innerhalb der vier nordischen Länder Schweden, Dänemark, Finnland und Norwegen und besonders in Schweden für die Zukunft die Stellung aufrecht zu erhalten und weiter auszubauen, die die reine Mathematik in diesen Ländern jetzt einnimmt und dadurch zugleich dem Beitrag dieser Länder zum höchsten Gedankenleben Achtung und gerechte Einschätzung ausserhalb der Grenzen des Nordens zu erwerben.

Extrait du testament dressé et signé le 16 mars 1916 par
G. MITTAG-LEFFLER et SIGNE MITTAG-LEFFLER, dée AF LINDFORS.

Nous soussignés, modifiant le testament mutuel dressé entre nous le 6 janvier 1883, déclarons ici notre dernière volonté, qui est de léguer tous nos biens, pour lui revenir après notre mort à tous deux, à une fondation qui prendra le nom de:

Institut mathématique des époux Mittag-Leffler.

Cet institut aura pour tâche de conserver aux mathématiques pures et de développer encore, dans les quatre pays scandinaves, Suède, Danemark, Finlande et Norvège, mais tout particulièrement en Suède, la position qu’elles y occupent aujourd’hui, comme aussi de faire connaître et estimer à sa juste valeur, en dehors de leurs frontières, l’apport de ces pays dans la sphère la plus haute de la vie de l’esprit.
There should be a foundation and a scientific director with tasks:

- to maintain the house and the library, and keep the library up-to-date;
- to give grants to young promising Scandinavian mathematicians;
- to give prizes for really outstanding new results in mathematics;
- to subsidize further research positions at the institute.
Some institute directors

Niels Erik Nørlund  Torsten Carleman  Lennart Carleson

Carleman was director during ca. 1930–1949. Carleson started with special years: harmonic analysis, 1969–1970; noncommutative harmonic analysis, 1970–1971. Such thematic years or periods have continued until now.

These illustrations are listed on the next page, together with the corresponding page numbers in the book. As acknowledged on pp. ix, x of the book, most of the pictures are from the Institut Mittag-Leffler (IML) in Djursholm (see the table on next page for some further details).

Herewith I acknowledge to the Institut Mittag-Leffler, to Arild Stubhaug and to Springer-Verlag for having reproduced the pictures on my slides.
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