

# OP-SF NET – Volume 22, Number 4 – July 15, 2015

The Electronic News Net of the  
SIAM Activity Group on Orthogonal Polynomials and Special Functions

<http://math.nist.gov/opsf>

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Topics:

1. International Conference on Orthogonal Polynomials and  $q$ -series
2. Karl Liechty wins third Gábor Szegő Prize
3. OPSFA-13: A SIAM special function at NIST, by Willard Miller, Jr.
4. OPSFA-13: A short report, by Jacob Christiansen
5. OPSFA-13: A short report, by Tom Claeys
6. V Iberoamerican Workshop on Orthogonal Polynomials and Applications
7. Special session at AMS-EMS-SPM International meeting
8. Publishing of Table Errata in OP-SF Net
9. Errata and comments to HTF and ToIT, by Tom Koornwinder
10. Preprints in arXiv.org
11. About the Activity Group
12. Submitting contributions to OP-SF NET and SIAM-OPSF (OP-SF Talk)

## Calendar of Events:

### August 3–7, 2015

Symmetries of Discrete Systems and Processes, Děčín, Czech Republic

<http://sdsp.fjfidecin.cz/sdsp/tdc>

### August 9–14, 2015

Orthogonal and Multiple Orthogonal Polynomials, Oaxaca, Mexico

<http://www.birs.ca/events/2015/5-day-workshops/15w5022>

### August 10–14, 2015

ICIAM 2015 (International Congress on Industrial and Applied Mathematics),  
Beijing, China

<http://www.iciam2015.cn>

**August 26–28, 2015**

Symposium “The Real World is Complex” in honour of Christian Berg, in  
Copenhagen, Denmark  
<http://www.math.ku.dk/~henrikp/cb>

**September 10–12, 2015**

XIVth Annual Conference (ICSFA 2015) of Society for Special Functions &  
their Applications (SSFA), Amity University, Noida, India  
<http://www.ssfaIndia.webs.com/conf.htm>

**September 28–30, 2015**

International Conference on Analysis, Applications and Computations, In  
Memory of Lee Lorch, Fields Institute, Toronto, Canada  
<http://www.fields.utoronto.ca/programs/scientific/15-16/analysisapplications>

**October 4–30, 2015**

International Workshop and Latin–American School on Foundations of Complexity,  
Nonadditive Entropies and Nonextensive Statistical Mechanics, Rio de Janeiro, Brazil  
<http://www.cbpf.br/~complex>

**June 27 – July 1, 2016**

Abecederian of SIDE (ASIDE) 12 Summer School,  
Centre de Recherches mathématiques, Université de Montréal, Montréal, Quebec, Canada  
<http://www.crm.umontreal.ca/2016/ASIDE2016>

**July 3–9, 2016**

Symmetries and Integrability of Difference Equations 12,  
Hôtel Le Chanteclerc, Saint Adèle, Québec, Canada  
<http://www.crm.umontreal.ca/2016/SIDE12/index.php>

**July 11–15, 2016**

OPSF–S6 Summer School on Orthogonal Polynomials and Special Functions,  
American University, Washington D.C., USA  
<https://wis.kuleuven.be/events/OPSFA>

Topic #1 ——— OP – SF Net 22.4 ——— July 15, 2015

From: Frank Garvan ([fgarvan@ufl.edu](mailto:fgarvan@ufl.edu))

Subject: International Conference on Orthogonal Polynomials and  $q$ -series

A conference, **Orthogonal Polynomials and  $q$ -Series**, in honor of Professor Mourad Ismail’s 70<sup>th</sup> birthday, was held May 10–12, 2015 at the Radisson Hotel, Orlando, Florida. There were 41 speakers from Austria, Canada, China, France, Germany, Hong Kong, the Netherlands, Qatar, Spain, U.S.A, and the United Arab Emirates. The plenary speakers were: Richard Askey, *Some of what I owe to Mourad Ismail and some elementary inequalities, both old and new*; and Dennis Stanton, *A small slice of Mourad’s work*.

The keynote speakers were: George Andrews, *Bressoud’s easy proof of the Rogers-Ramanujan identities and Bressoud polynomials*; Willi Freeden, *Euler summation and Shan-*

*non sampling*; Frank Garvan, *Transformation properties of Dyson's rank function*; Erik Koelink, *An explicit family of matrix-valued orthogonal polynomials in the  $q$ -Askey scheme*; Tom Koornwinder *Fractional integral and generalized Stieltjes transforms for hypergeometric functions as transmutation operators*; Victor Moll, *A collection of questions coming from the evaluation of integrals*; Paul Nevai, *Some inequalities in approximation theory*; Luc Vinet, *A  $q$ -generalization of the Bannai-Ito polynomials and the quantum superalgebra  $\mathfrak{osp}_q(1|2)$* ; Roderick Wong, *Asymptotics and orthogonal polynomials*; and Ruimin Zhang, *On the Fourier transform and  $q$ -spherical function*. There were also parallel sessions of contributed talks. On Sunday evening there was a reception. On Monday morning there was a group photo.



On Monday evening there was a banquet in honor of Mourad Ismail. Many touching reminiscences of Mourad were made. The conference ended at lunchtime on Tuesday after talks by Paul Nevai and Luc Vinet. The conference was supported by the University of Central Florida (College of Science and Department of Mathematics), and the King Saud University.

## Topic #2 ——— OP – SF Net 22.4 ——— July 15, 2015

From: Walter Van Assche ([Walter.VanAssche@wis.kuleuven.be](mailto:Walter.VanAssche@wis.kuleuven.be))

Subject: Karl Liechty wins third Gábor Szegő Prize

The Gábor Szegő prize 2015 was awarded to Karl Liechty (DePaul University, Chicago) during the 13th symposium on Orthogonal Polynomials, Special Functions and Application (OPSFA-13), which was held at NIST, Gaithersburg, 1–5 June. Karl Liechty received the prize “For his original work in the asymptotic analysis of orthogonal polynomials arising in models from statistical mechanics, in particular the six-vertex model and a model of non-intersecting random paths.”

Karl Liechty obtained his PhD in mathematics in 2010 from Indiana University–Purdue University Indianapolis where his PhD advisor was Pavel Bleher. After his PhD he spent one semester at the Mathematical Sciences Research Institute (MSRI) in Berkeley and then was a postdoc for three and a half years at the University of Michigan. In 2014 he joined the faculty at DePaul University and presently he is Assistant Professor in their Department of Mathematical Sciences.

Karl Liechty was nominated for the Gábor Szegő prize for the following papers

1. P. Bleher, K. Liechty: *Exact solution of the six-vertex model with domain wall boundary conditions. Ferroelectric phase*, *Comm. Math. Phys.* **286** (2009), 777–801.

2. P. Bleher, K. Liechty: *Exact solution of the six-vertex model with domain wall boundary conditions. Critical line between ferroelectric and disordered phases*, J. Stat. Phys. **134** (2009), 463–485.
3. P. Bleher, K. Liechty: *Exact solution of the six-vertex model with domain wall boundary conditions. Antiferroelectric phase*, Comm. Pure Appl. Math. **63** (2010), 779–829.
4. K. Liechty: *Nonintersecting Brownian motions on the half-line and discrete Gaussian orthogonal polynomials*, J. Stat. Phys. **147** (2012), 582–622.

More recently he has written a book with Pavel Bleher on *Random Matrices and the Six-Vertex Model* (CRM Monograph Series, vol. 32, 2014, Amer. Math. Soc., Providence RI, 224 pp.).

In these papers and in the book, essential and non-trivial use is made of orthogonal polynomials. These novel applications of orthogonal polynomials not only helped to solve important problems in statistical physics, but also gave deeper insight into the asymptotic behavior of orthogonal polynomials of various kinds. These contributions to the asymptotic theory of orthogonal polynomials are original and profound. The fact that the results are motivated by physical problems is vital for the development of the area of orthogonal polynomials and special functions.



The Gábor Szegő prize is awarded by the SIAM Activity Group on Orthogonal Polynomials and Special Functions (SIAG/OPSF) and consists of a plaque and a certificate. The recipient is invited to give a plenary lecture at the OPSFA meeting and SIAM pays the registration fee, travel and accommodation.

Karl's plenary talk was on "*Tacnode kernels and Lax systems for the Painlevé II equation.*" The talk dealt with a special determinantal process studied by several groups of researchers around 2010. The kernel defining the process can be described by a Lax system for the Painlevé II differential equation involving a  $4 \times 4$  matrix. Karl and his co-author Dong Wang showed how this Lax system is related to the  $2 \times 2$  Lax pair for Painlevé II studied by Flaschka and Newell in 1980. This resulted in new formulas for various tacnode kernels in random matrix theory.

The Gábor Szegő prize selection committee consisted of Walter Van Assche (chair), Peter Clarkson, Charles Dunkl, Jeff Geronimo, and Kerstin Jordaan. The next Gábor Szegő prize will be awarded in 2017 at the OPSFA-14 conference which will be held at the University of Kent, Canterbury, UK.

## Topic #3 ——— OP – SF Net 22.4 ——— July 15, 2015

From: Willard Miller, Jr. ([miller@ima.umn.edu](mailto:miller@ima.umn.edu))

Subject: OPSFA-13: A SIAM special function at NIST, by Willard Miller, Jr.

The 13th International Symposium on Orthogonal Polynomials, Special Functions and Applications (OPSFA13) was held June 1–5, 2015 at the National Institute of Standards and Technology (NIST), headquarters in Gaithersburg, Maryland. The symposium was sponsored by the SIAM Activity Group on Orthogonal Polynomials and Special Functions (SIAG/OPSF), the NIST Applied and Computational Mathematics Division, and supported by the NSF and NIST. NIST was formerly called the National Bureau of Standards (NBS). It was a particularly appropriate site to host the Symposium (the first in the US), because of NBS/NIST's leading role in developing handbooks and other reference material that provide basic formulas and notation for special functions as well as numerical tables and software.

There is no technical definition of special functions. Basically, they are useful functions, those that occur so frequently in the sciences that it becomes imperative to collect them in handbooks describing their properties and codifying notation. Most of these functions are the “special functions of mathematical physics” (those that arise as solutions of 2nd order linear differential and difference equations in chemistry, physics and engineering, such as via separation of variables methods and as families of orthogonal polynomials), e.g., hypergeometric and  $q$ -hypergeometric functions. More recently other functions, such as Painlevé functions (satisfying 2nd order nonlinear differential equations) and exceptional polynomials (the Askey scheme and beyond) have risen to handbook status.

NBS and NIST have played an invaluable role in codifying and disseminating information about special functions. Work on the NBS tables started in 1938, culminating in the 1964 publication of the NBS handbook (edited by Milton Abramowitz and Irene A. Stegun). The earlier NBS tables were largely numerical, oriented toward practical computations. In recognition of the emerging predominance of mathematical software, the 1964 handbook focused more on formulas, with tables of numerical values filling less than half its pages. Walter Gautschi and the late Frank Olver played major roles in developing the material for the handbook. There are likely over a million copies in print and it remains the most highly cited mathematics publication of NIST.

The 1964 handbook was succeeded in 2010 by the Digital Library of Mathematical Functions (DLMF), which is available as a handbook and also online (<http://dlmf.nist.gov>). The DLMF contains almost no tables of numbers and more than twice as many formulas as the old handbook. The online version is continually updated; it imparts information about the behavior of these functions through interactive graphics. Frank Olver provided the subject matter leadership for this project; (he was the founding editor of the SIAM Journal on Mathematical Analysis (SIMA), 1970, and for years SIMA was one of the most prestigious journals that covered special functions, before there was a change of editorial policy). The other editors are Daniel W. Lozier, Ronald F. Boisvert, and Charles W. Clark. Part of the meeting was devoted to assessing the history and continuing updates of the project and discussing plans for the future.

The OPSFA International Symposia date back to 1984 with the first meeting in Bar-Le-Duc, France. The invited lectures then were by the luminaries J. Dieudonné: *Fractions continuées et polynômes orthogonaux dans l'oeuvre de E. N. Laguerre*, W. Hahn : *Ueber*

Orthogonalpolynome die linearen Differenzgleichungen genuegen, G. Andrews and R. Askey: Classical orthogonal polynomials, and W. Gautschi: Some new applications of orthogonal polynomials. The present 13th meeting was the first to be held in the USA. It was truly international: There were more than 200 registrants from 35 countries, more than half from outside the US.

The community of researchers who use special functions is divided into two overlapping categories, those whose main interest is the special functions themselves and their properties, and those that are motivated by other branches of the sciences but encounter special functions and orthogonal polynomials in these pursuits. This meeting was no exception. There were 10 plenary talks, 4 on topics focusing on the core body of the theory of special functions and orthogonal polynomials: 1) Vector-valued nonsymmetric and symmetric Jack and Macdonald polynomials (Charles Dunkl), 2) Eigenvalue of large Hankel and Toeplitz matrices (Mourad Ismail), 3) Asymptotic and numerical aspects of special functions (Nico Temme), 4) Multivariate orthogonal polynomials and modified moment functionals (Teresa Pérez) and 6 where the motivation came from other research areas: 1) On the asymptotic behavior of a log gas in the bulk scaling limit in the presence of a varying external potential (Percy Deift) [random matrices, Riemann–Hilbert problems, Airy functions], 2) Hypergeometric series (Wadim Zudilin) [number theory, Riemann zeta function], 3) Integrable probability and the role of Painlevé functions (Craig Tracy) [random matrices], 4) Limits of orthogonal polynomials and contractions of Lie algebras (Sarah Post) [symmetries in mathematical physics], 5) The Laguerre–Polya Class (Olga Holtz) [polynomials with real roots, optimization theory] 6) Orthogonal Polynomials and the 2–Species ASEP (Lauren Williams) [models of particles on lattices].

Every two years the Activity Group (SIAG/OPSF), joint with SIAM and the OPSFA symposium series leadership, awards the Gábor Szegő Prize to an early-career researcher for outstanding research contributions in the area of orthogonal polynomials and special functions. This year the prize was awarded to Karl Liechty, who delivered the Gábor Szegő Prize lecture: Tacnode Kernels and Lax systems for the Painlevé II equation.

In addition to contributed talks there were 21 minisymposia with invited talks, 11 in the core body of special function theory: [Numerical and symbolic computational aspects of special functions, orthogonal polynomials of several variables, the legacy of Ramanujan, semiclassical orthogonal polynomials, inequalities and special functions, asymptotics of orthogonal polynomials, orthogonal polynomials and moment problems, multiple orthogonal polynomials], and 10 that were more heavily influenced and motivated by other research areas: [Number theory and special functions (especially connections with the Riemann zeta function), potential theory and applications to orthogonal polynomials and minimal energy (Riemann–Hilbert approach), Riemann–Hilbert problems (applications to differential equations and random matrix theory), Sobolev orthogonal polynomials (boundary value problems for PDEs), aspects of Painlevé equations, symmetry and special functions, orthogonal polynomials of the discrete variables on lattices, Szegő's theorem and its generalizations (connects asymptotics of Toeplitz matrices and their coefficients)]. In addition there was a minisymposium on digital mathematics libraries which focused on the NIST [DLMF](#) and [DRMF](#) projects, [WDML](#), [MathSciNet](#), [zbMATH](#), [DDMF](#) and the Mathematica Computable Library of Special Function Identities.

The SIAM Activity Group on Orthogonal Polynomials and Special Functions (SIAG/OPSF) was organized as an official SIAM Activity Group in 1990, under the leadership of Charles Dunkl. The newsletter (<http://math.nist.gov/opsf>) of the Activity Group started in 1993. The Gábor Szegő Prize was launched in 2010, thanks to Francisco Marcellán, who was

chairing the group at that time.

In this era of machine computation and computer simulation, special functions and orthogonal polynomials remain vibrant areas of research for reasons beyond the intrinsic beauty of the theory. The construction of realistic mathematical models of real world systems which can be solved analytically and explicitly with adjustable parameters through the use of special functions, plays a vital role in the understanding of the structure of these systems. This role was clearly demonstrated at OPSFA-13.

Topic #4 ——— OP – SF Net 22.4 ——— July 15, 2015

From: Jacob Christiansen ([stordaljc@gmail.com](mailto:stordaljc@gmail.com))  
Subject: OPSFA-13: A short report, by Jacob Christiansen

This was my seventh OPSFA (since the one in Rome in 2001) and I very much looked forward to it. For the first time ever, the meeting was held in the US and I hence expected to meet many of my American colleagues. The meeting was well-attended, with more than 150 participants ranging from first-timers to Askey and Chihara.

I enjoyed many of the plenary talks, covering different topics and yet sometimes also linked. They were all well-presented, I think. The bulk of the remaining programme was divided into mini-symposia. With 5-6 or even 7 sessions at the same time, one had to pick and choose. I followed MS3 on Number Theory and Special Functions to catch up on  $q$ -series and Rogers-Ramanujan type identities and I was impressed by the vast developments for Aspects of Painlevé Equations (MS13-18-23). I enjoyed the session on Asymptotics of Orthogonal Polynomials (MS38-40) but missed parts of it due to overlap with my own talk. (Perhaps the technician also had questions here...)

If I should point at two talks that I found particularly interesting, it would be Eichinger's (CP1) explaining how to generalize the Killip-Simon theorem and Swiderski's (MS30) solving a conjecture of Chihara. It was great to see graduate students attack hard problems within our field.

Perhaps a half-day excursion to Washington would have been a nice break in the dense week. But overall an exciting and diverse scientific programme.

Topic #5 ——— OP – SF Net 22.4 ——— July 15, 2015

From: Tom Claeys ([tom.claeys@uclouvain.be](mailto:tom.claeys@uclouvain.be))  
Subject: OPSFA-13: A short report, by Tom Claeys

The 13th OPSFA meeting took place at the National Institute of Standards and Technology in Gaithersburg, Maryland, from June 1-5. The participants were brought every day from the hotel area to the highly secured NIST and back with a shuttle bus.

The schedule was very dense with 2-3 plenary talks every day, and many minisymposia and contributed sessions. Because of the parallel sessions, it was often hard to choose which talks to attend. I mostly attended the minisymposia on "Riemann-Hilbert problems: Orthogonal Polynomials and Random Matrix Theory," "Multiple Orthogonal Polynomials," and "Asymptotics of Orthogonal Polynomials," in which numerous interesting talks were given. On the other hand, I regret that I could not attend more talks in the

minisymposium on “Aspects of Painlevé Equations.”

I also enjoyed the plenary talk by Craig Tracy, in which he gave an overview of results in integrable probability and stochastic growth models, in relation with Painlevé equations. The plenary talk by Percy Deift, who discussed asymptotics for sine kernel Fredholm determinants, was also very interesting.

Karl Liechty was awarded the Gábor Szegő prize, and in the prize lecture he explained some of his nice recent results about different Lax pairs for the Painlevé II equation: in particular he obtained an explicit relation between the standard Flaschka–Newell Lax pair and a  $4 \times 4$  Lax pair.

## Topic #6 ——— OP – SF Net 22.4 ——— July 15, 2015

From: Luis E. Garza (Universidad de Colima, México) ([garzaleg@gmail.com](mailto:garzaleg@gmail.com))  
Subject: V Iberoamerican Workshop on Orthogonal Polynomials and Applications

The Iberoamerican Workshop on Orthogonal Polynomials and Applications took place at the Institute of Mathematics (IMATE) of the Universidad Nacional Autónoma de México (UNAM), in Mexico City, on the second week of June. This event was organized by Baltazar Aguirre Hernández, Abdón Choque–Rivero, Manuel Domínguez de la Iglesia, and Luis E. Garza. It is the fifth edition of this event organized by the Iberoamerican Group of Orthogonal Polynomials and Applications (GIBPOA).

The workshop program consisted of two short courses intended for students, six plenary talks and twelve short talks. The topics of the short courses were multivariate orthogonal polynomials (by Teresa Pérez, Universidad de Granada) and exceptional polynomials (by David Gómez–Ullate, ICMAT and Universidad Complutense de Madrid). Both courses were nicely structured and presented by the expositors, and captured the attention of the audience. Some connections with other topics on orthogonal polynomials and several open problems were discussed.

The plenary talks included: Natig Atakishiyev: On a discrete number operator and its eigenvectors associated with the 5D discrete Fourier transform; Jesús Muciño Raymundo: Polynomial determined by their critical points; Luis Verde–Star: Matrices,  $q$ –orthogonal polynomials and the Askey’s scheme; Juan Tirao: Reducibility of matrix weights; Daniel Malacara: Zernike polynomials to represent wavefront aberration; and Eric Campos Cantón: Polynomials for the generation of chaotic dynamics.

The short talks were presented by professors and some students. In fact, one of the goals of the workshop is to provide a forum where Latin American students who start working on orthogonal polynomials and related topics can present the results of their theses. The attendance consisted of about 20 professors and 30 students from Argentina, Brazil, Colombia, Cuba, Mexico and Spain, which had the opportunity to get to know each other during the five days of the workshop.

The great work of the local organizer Manuel Domínguez de la Iglesia, the friendly environment, and the interesting presentations and discussions made this workshop a memorable one.

Complete information of the event, as well as links to the pdf files of the presentations can be found at <http://paginas.matem.unam.mx/eibpoa2015/index.php/en>.

## Topic #7 ——— OP – SF Net 22.4 ——— July 15, 2015

From: Kerstin Jordaan ([kjordaan@up.ac.za](mailto:kjordaan@up.ac.za))

Subject: Special session at AMS–EMS–SPM International meeting

The AMS–EMS–SPM conference was a joint corporation between the American Mathematical Society (AMS), the European Mathematical Society (EMS) and the Portuguese Mathematical Society (SPM). The AMS liaises with a host society from another country on an annual basis to organise a meeting with the aim of linking together members of the mathematical communities from different countries and continents with those of the US.

The latest meeting was held on the campus of the University of Porto in the UNESCO world heritage city of Porto in Portugal from Wednesday 10 June to Saturday 13 June, 2015. The scientific scope of this meeting ranged from plenary talks of general interest to a variety of special sessions focusing on current research in specific areas.

There were invited addresses by distinguished lecturers, including Annette Huber from Albert–Ludwigs–Universität, André Neves from the Imperial College London, Rui Loja Fernandes from the University of Illinois, Gigliola Staffilani from MIT, Irene Fonseca from the Carnegie Mellon University, Marcelo Viana from the Instituto de Matemática Pura e Aplicada in Rio de Janeiro and Sylvia Serfaty from the Université Pierre et Marie Curie Paris 6 as the EMS Distinguished Speaker. An evening public lecture at Casa da Música (the main house of musical performances of Porto) by Marcus du Sautoy from the University of Oxford on The Secret Mathematicians was followed by a musical recital by an orchestra from the Faculty of Engineering of the University of Porto. The social program also included a reception and a banquet.

This conference was arranged into morning, afternoon and early evening sessions with the special sessions running in parallel during the two earlier sessions. The plenary talks were then held in the last session. There were 53 special sessions devoted to different themes. The 47<sup>th</sup> special session on Orthogonal Polynomials and Integral Equations was organised by Peter Clarkson, Ana Loureiro and Walter van Assche. More detailed information on the talks presented is available at:

[http://aep-math2015.spm.pt/sites/default/files/program\\_special\\_sessions.pdf](http://aep-math2015.spm.pt/sites/default/files/program_special_sessions.pdf)

This conference had more than 700 presenters and over 1000 participants. Organising a conference of this scope must have been a huge undertaking. The local organisers handled it with aplomb and everything ran extremely smoothly.

## Topic #8 ——— OP – SF Net 22.4 ——— July 15, 2015

From: OP–SF Net Editors

Subject: Publishing of Table Errata in OP–SF Net

The AMS journal Mathematics of Computation ([MCOM](#)) used to publish Table Errata. Several years ago, MCOM made the decision to no longer publish Table Errata. The purpose of publishing Table Errata in MCOM was so that there would exist a permanent record of the Table Errata. This way, one could easily look up errata in the permanent record and one would always be able to locate them. This has proved very useful over the years. For instance, whenever available, the DLMF refers to Table Errata for OPSF books in MCOM.

With the recent decision by MCOM, there is currently no place to permanently store Table Errata. This problem has been temporarily solved by publishing Table Errata for OPSP books in OP-SF NET. The option of a permanent errata page on the SIAM website is being investigated.

## Topic #9 ——— OP – SF Net 22.4 ——— July 15, 2015

From: Tom Koornwinder ([T.H.Koornwinder@uva.nl](mailto:T.H.Koornwinder@uva.nl))

Subject: Errata and comments to HTF and ToIT, by Tom Koornwinder

These are comments and possibly not yet published errata for the volumes:

A. Erdélyi *et al.*, **Higher Transcendental Functions, Vols. 1, 2, 3**, McGraw-Hill, 1953, 1953, 1955, and **Tables of Integral Transforms, Vols. 1, 2**, McGraw-Hill, 1954, 1954.

For further errata, see also the lists of errata which are included in the volumes, and the errata collected, for instance, by H. van Haeringen and L. P. Kok in Math. Comp. 41 (1983), 778–780 (see <http://www.jstor.org/stable/2007718>). Note that all previous collected table errata for these volumes can be found in the DLMF ⓘ-boxes for the references of A. Erdélyi *et al.* at <http://dlmf.nist.gov/bib/E>.

### Higher Transcendental Functions, Vol. 1

**p.104, 2.8(54)**: On the left replace  $3a + 5/6$  (the third argument of  $F$ ) by  $2a + 5/6$ . See the correct formula (15.4.32) – <http://dlmf.nist.gov/15.4#iii>. For the proof and an observation of the error in Higher Transcendental Functions, click there on the information on the right of the subsection header.

**p.115, 2.12(6)**: The side condition on the parameters should be  $\operatorname{Re} c > \operatorname{Re} b > 0$ .

**p.144, 3.4(8)**: On the right, after the equality sign, replace  $i\pi$  by  $-i\pi$  (observed by E. Diekema; see Ch. IV, (99) in L. Robin, *Fonctions sphériques de Legendre et fonctions sphéroïdales, Tome II*, Gauthier-Villars, 1958).

**p.146, after 3.5(3)**: For the convergence of both series require additionally that  $\operatorname{Re} \mu < \frac{1}{2}$ .

**p.175, 3.15(4)**: This formula is valid for  $z \in \mathbb{C} \setminus (-\infty, 1]$ . For  $x \in (-1, 1)$  the formula remains valid if we replace  $(z^2 - 1)^{\frac{1}{4} - \frac{1}{2}\nu}$  by  $(1 - x^2)^{\frac{1}{4} - \frac{1}{2}\nu}$  and  $P_{n+\nu-\frac{1}{2}}^{\frac{1}{2}-\nu}$  by  $P_{n+\nu-\frac{1}{2}}^{\frac{1}{2}-\nu}$ :

$$C_n^\nu(x) = 2^{\nu-\frac{1}{2}} \frac{\Gamma(n+2\nu)\Gamma(\nu+\frac{1}{2})}{\Gamma(2\nu)\Gamma(n+1)} (1-x^2)^{\frac{1}{4}-\frac{1}{2}\nu} P_{n+\nu-\frac{1}{2}}^{\frac{1}{2}-\nu}(x),$$

where  $P_\nu^\mu$  is defined in 3.4(1).

**p.230, 5.8(3)**: In the integrand the exponent of  $(1-u-v)$  should be  $\gamma - \beta - \beta' - 1$ . See the correct formula (16.15.3) – <http://dlmf.nist.gov/16.15.E3>, which currently refers to Erdélyi *et al.* (1953a, §5.8) (see text below (16.15.4)) without observing the error in 5.8(3).

### Higher Transcendental Functions, Vol. 2

**p.174, 10.9(8)**: In the formula for  $K_n$  insert a factor  $n!$  on the right.

**p.179, 10.10(5)**: In the formula for  $C_n$  delete the minus sign on the right.

**p.185, 10.11(16):** But for  $n = 0$  and  $z_m = T_m$  we have  $z_1(x) = xz_0(x)$ .

**p.188, 10.12(2):** The formula for  $r_n$  should read:  $r_n = -n(n + \alpha)$ .

### Tables of Integral Transforms, Vol. 1

**p.38, 1.10(5):** On the left, replace the expression for  $f(x)$  ( $0 < x < 1$ ) by  $(1 - x)^\nu(1 + x)^\mu P_{2n}^{(\nu, \mu)}(x) + (1 + x)^\nu(1 - x)^\mu P_{2n}^{(\mu, \nu)}(x)$ .

**p.38, 1.10(6):** On the left, replace the expression for  $f(x)$  ( $0 < x < 1$ ) by  $(1 - x)^\nu(1 + x)^\mu P_{2n+1}^{(\nu, \mu)}(x) - (1 + x)^\nu(1 - x)^\mu P_{2n+1}^{(\mu, \nu)}(x)$ .

On the right, replace  $(-1)^{n+1}$  by  $(-1)^n$ .

**p.94, 2.10(6):** On the left, replace the expression for  $f(x)$  ( $0 < x < 1$ ) by  $(1 - x)^\nu(1 + x)^\mu P_{2n}^{(\nu, \mu)}(x) - (1 + x)^\nu(1 - x)^\mu P_{2n}^{(\mu, \nu)}(x)$ .

**p.95, 2.10(7):** On the left, replace the expression for  $f(x)$  ( $0 < x < 1$ ) by  $(1 - x)^\nu(1 + x)^\mu P_{2n+1}^{(\nu, \mu)}(x) + (1 + x)^\nu(1 - x)^\mu P_{2n+1}^{(\mu, \nu)}(x)$ .

On the right, replace  $(-1)^{n+1}$  by  $(-1)^n$ .

**p.123, 3.3(4):** On the left, replace  $P_n^{(\nu, \nu)}$  by  $P_n^{(\nu, \mu)}$ .

This formula implies 1.10(5), 1.10(6), 2.10(6) and 2.10(7).

### Tables of Integral Transforms, Vol. 2

**p.399, 20.2(6):** On the right replace  $(1 - z)^\sigma$  by  $(1 - z)^{-\sigma}$ .

This formula is correctly reproduced in Gradshteyn & Ryzhik, sixth ed., (7.512.9).

Topic #10 ——— OP – SF Net 22.4 ——— July 15, 2015

From: OP–SF Net Editors

Subject: Preprints in arXiv.org

The following preprints related to the fields of orthogonal polynomials and special functions were posted or cross-listed to one of the subcategories of arXiv.org during May and June 2015.

<http://arxiv.org/abs/1505.00029>

Differentiable positive definite functions on two-point homogeneous spaces

V. S. Barbosa, V. A. Menegatto

<http://arxiv.org/abs/1505.00084>

On the BMV conjecture for  $2 \times 2$  matrices and the exponential convexity of the function  $\cosh(\sqrt{at^2 + b})$

Victor Katsnelson

<http://arxiv.org/abs/1505.00742>

Nevanlinna extremal measures for polynomials related to  $q^{-1}$ -Fibonacci polynomials

František Štampach

<http://arxiv.org/abs/1505.00885>

Autonomous limit of 4-dimensional Painlevé-type equations and degeneration of curves

of genus two  
Akane Nakamura

<http://arxiv.org/abs/1505.01431>

Growing the Digital Repository of Mathematical Formulae with Generic LaTeX Sources  
Howard S. Cohl, Moritz Schubotz, Marjorie A. McClain, Bonita V. Saunders, Cherry Y. Zou,  
Azeem S. Mohammed, Alex A. Danoff

<http://arxiv.org/abs/1505.01472>

Directional convexity and characterizations of Beta and Gamma functions  
Martin Himmel, Janusz Matkowski

<http://arxiv.org/abs/1505.01528>

On polynomials associated with an Uvarov modification of a quartic potential Freud-like  
weight  
Alejandro Arceo, Edmundo J. Huertas, Francisco Marcellán

<http://arxiv.org/abs/1505.01555>

Some physical applications of generalized Lambert functions  
István Mező, Grant Keady

<http://arxiv.org/abs/1505.02143>

On perturbed orthogonal polynomials on the real line and the unit circle via Szegő's  
transformation  
K. Castillo, F. Marcellán, J. Rivero

<http://arxiv.org/abs/1505.02178>

Expansions of the solutions of the confluent Heun equation in terms of the incomplete  
Beta and the Appell generalized hypergeometric functions  
C. Leroy, A.M. Ishkhanyan

<http://arxiv.org/abs/1505.02179>

On Some  $q$ -Analogues of the Natural Transform and Further Investigations  
S. K. Q. Al-Omari

<http://arxiv.org/abs/1505.02604>

Asymptotics of Chebyshev Polynomials, I. Subsets of  $\mathbb{R}$   
Jacob S. Christiansen, Barry Simon, Maxim Zinchenko

<http://arxiv.org/abs/1505.02958>

Sharp Pitt inequality and logarithmic uncertainty principle for Dunkl transform in  $L^2$   
Dmitry Gorbachev, Valery Ivanov, Sergey Tikhonov

<http://arxiv.org/abs/1505.03993>

Hermite-Padé approximants for a pair of Cauchy transforms with overlapping symmetric  
supports  
Alexander I. Aptekarev, Walter Van Assche, Maxim L. Yattselev

<http://arxiv.org/abs/1505.05468>

Comments on "New generating relations for products of two Laguerre polynomials"  
Xiaoxia Wang, Arjun K. Rathie

<http://arxiv.org/abs/1505.06515>

Uncovering functional relationships at zeros with special reference to Riemann's Zeta Function

M. L. Glasser, Michael Milgram

<http://arxiv.org/abs/1505.06635>

An Orthogonality Property of the Legendre Polynomials

Len Bos, Akil Narayan, Norm Levenberg, Federico Piazzon

<http://arxiv.org/abs/1505.07134>

New Laplace transforms for the generalized hypergeometric functions  ${}_2F_2$  and  ${}_3F_3$

Xiaoxia Wang, Arjun K. Rathie

<http://arxiv.org/abs/1505.07788>

Extreme zeros in a sequence of para-orthogonal polynomials and bounds for the support of the measure

A. Sri Ranga, Daniel O. Veronese

<http://arxiv.org/abs/1505.01552>

Koshliakov kernel and identities involving the Riemann zeta function

Atul Dixit, Nicolas Robles, Arindam Roy, Alexandru Zaharescu

<http://arxiv.org/abs/1505.02271>

From conformal group to symmetries of hypergeometric type equations

Jan Dereziński, Przemysław Majewski

<http://arxiv.org/abs/1505.04762>

Expected number of real zeros for random Freud orthogonal polynomials

Igor E. Pritsker, Xiaoju Xie

<http://arxiv.org/abs/1505.05926>

Special Functions of Hypercomplex Variable and Discrete Electromagnetic Schrödinger Operators

Nelson Faustino

<http://arxiv.org/abs/1505.06192>

An invariant class of Hermite type multivariate polynomials for the Wigner transform

Helge Dietert, Johannes Keller, Stephanie Troppmann

<http://arxiv.org/abs/1506.00087>

2-iterated Sheffer polynomials

Subuhi Khan, Mumtaz Riyasat

<http://arxiv.org/abs/1506.00444>

The third, fifth and sixth Painlevé equations on weighted projective spaces

Hayato Chiba

<http://arxiv.org/abs/1506.01672>

Dunkl completely monotonic functions

Jamel El Kamel, Khaled Mehrez

<http://arxiv.org/abs/1506.02502>

On the zeros of the Pearcey integral and a Rayleigh-type equation  
Gerardo Hernández-del-Valle

<http://arxiv.org/abs/1506.06545>

Hamiltonian system for the elliptic form of Painlevé VI equation  
Zhijie Chen, Ting-Jung Kuo, Chang-Shou Lin

<http://arxiv.org/abs/1506.07394>

On the  $(p, q)$ -Gamma and the  $(p, q)$ -Beta functions  
P. Njionou Sadjang

<http://arxiv.org/abs/1506.07803>

Tridiagonalization of the hypergeometric operator and the Racah-Wilson algebra  
Vincent X. Genest, Mourad E. H. Ismail, Luc Vinet, Alexei Zhedanov

<http://arxiv.org/abs/1506.08708>

Multivariate orthogonal Laurent polynomials and integrable systems  
Gerardo Ariznabarreta, Manuel Mañas

<http://arxiv.org/abs/1506.09159>

Certain Inequalities Involving the  $q$ -Deformed Gamma Function  
Kwara Nantomah, Edward Prempeh

<http://arxiv.org/abs/1506.09160>

Reductions of particular hypergeometric functions  ${}_3F_2(a, a + 1/3, a + 2/3; p/3, q/3; \pm 1)$   
Mark W. Coffey

<http://arxiv.org/abs/1506.02137>

Two closed forms for the Bernoulli polynomials  
Feng Qi, Robin J. Chapman

<http://arxiv.org/abs/1506.07084>

Complex Hermite functions as Fourier-Wigner transform  
Fatima Agorram, Arij Benkhadra, Amal El Hamyani, Allal Ghanmi

<http://arxiv.org/abs/1506.07379>

Generalized Hurwitz matrices, generalized Euclidean algorithm, and forbidden sectors of the complex plane  
Olga Holtz, Sergey Khrushchev, Olga Kushel

<http://arxiv.org/abs/1506.07382>

Conformable Fractional Bessel Equation and Bessel Functions  
Ahmet Gökdoğan, Emrah Ünal, Ercan Çelik

<http://arxiv.org/abs/1506.07386>

Some relations involving the higher derivatives of the Riemann zeta function  
Donal F. Connon

<http://arxiv.org/abs/1506.07393>

Further Inequalities Associated with the Classical Gamma Function  
Kwara Nantomah

<http://arxiv.org/abs/1506.03434>

Trajectories of quadratic differentials for Jacobi polynomials with complex parameters  
A. Martinez–Finkelshtein, P. Martinez–Gonzalez, F. Thabet

<http://arxiv.org/abs/1506.03651>

Recurrence Relations for Exceptional Hermite Polynomials  
D. Gomez–Ullate, A. Kasman, A.B.J. Kuijlaars, R. Milson

<http://arxiv.org/abs/1506.03740>

Sharp bounds for cumulative distribution functions  
Javier Segura

<http://arxiv.org/abs/1506.04682>

Connection coefficients for classical orthogonal polynomials of several variables  
Plamen Iliev, Yuan Xu

<http://arxiv.org/abs/1506.05306>

Representations for the parameter derivatives of some Koornwinder polynomials  
Rabia Aktas

<http://arxiv.org/abs/1506.07970>

Moments of  $q$ -Normal and conditional  $q$ -Normal distributions  
Paweł J. Szabłowski

<http://arxiv.org/abs/1506.07612>

The Zagier polynomials. Part III: Asymptotics and Exact formulas  
Atul Dixit, M. Lawrence Glasser, Victor H. Moll, Christophe Vignat

<http://arxiv.org/abs/1506.04283>

Series Representation of Modified Bessel Functions and Its Application in AF Cooperative systems  
Mehdi M. Molu

<http://arxiv.org/abs/1506.03138>

Inclusion of generalized Bessel functions in the Janowski class  
Saiful R. Mondal, Al Dhuain Mohammed

<http://arxiv.org/abs/1505.00355>

Multiplier sequences, classes of generalized Bessel functions and open problems  
George Csordas, Tamás Forgács

<http://arxiv.org/abs/1506.07267>

A generalization of the Sears–Slater transformation and elliptic Lagrange interpolation of type  $BC_n$   
Masahiko Ito, Masatoshi Noumi

<http://arxiv.org/abs/1506.02755>

Bounded Littlewood identities

Eric M. Rains, S. Ole Warnaar

<http://arxiv.org/abs/1506.00503>

Hypergeometric polynomials are optimal  
D.V.Bogdanov, T.M.Sadykov

<http://arxiv.org/abs/1505.06697>

New hypergeometric connection formulae between Fibonacci and Chebyshev polynomials  
W. M. Abd-Elhameed, Y. H. Youssri, N. El-Sissi, M. Sadek

<http://arxiv.org/abs/1505.02900>

Finite hypergeometric functions  
Frits Beukers, Henri Cohen, Anton Mellit

<http://arxiv.org/abs/1505.00610>

Correlation kernels for sums and products of random matrices  
Tom Claeys, Arno B. J. Kuijlaars, Dong Wang

<http://arxiv.org/abs/>

Solutions to the Incomplete Toronto Function and Incomplete Lipschitz-Hankel Integrals  
Paschalis C. Sofotasios, Steven Freear

<http://arxiv.org/abs/1505.06676>

A note on the  $-$ coefficients of the "tree Eulerian polynomial"  
Rafael S. González D'León

<http://arxiv.org/abs/1505.01948>

Some integral representations and limits for (products of) the parabolic cylinder function  
Dirk Veestraeten

<http://arxiv.org/abs/1505.00843>

Macdonald-Koornwinder moments and the two-species exclusion process  
Sylvie Corteel, Lauren Williams

<http://arxiv.org/abs/1506.07259>

From Random Matrix Theory to Coding Theory: Volume of a Metric Ball in Unitary Group  
Lu Wei, Renaud-Alexandre Pitaval, Jukka Corander, Olav Tirkkonen

<http://arxiv.org/abs/1506.06708>

On a particular form of a symmetric Pöschl-Teller potential  
Andrei Smirnov, Antonio Jorge Dantas Farias Jr

<http://arxiv.org/abs/1506.04925>

Dispersion and limit theorems for random walks associated with hypergeometric functions of type BC  
Michael Voit

<http://arxiv.org/abs/1506.07581>

Rigidity of Determinantal Point Processes with the Airy, the Bessel and the Gamma Kernel  
Alexander I. Bufetov

<http://arxiv.org/abs/1506.05734>

Orthogonal polynomials for the weakly equilibrium Cantor sets  
Gokalp Alpan, Alexander Goncharov

<http://arxiv.org/abs/1506.03312>

Regge symmetry and partition of Wigner 3-j or super 3-jS symbols: unknown properties  
Lionel Bréhamet

<http://arxiv.org/abs/1506.07580>

Moment Representations of the Exceptional  $X_1$ -Laguerre Orthogonal Polynomials  
Constanze Liaw, John Osborn

Topic #11 ——— OP – SF Net 22.4 ——— July 15, 2015

From: OP–SF Net Editors

Subject: About the Activity Group

The SIAM Activity Group on Orthogonal Polynomials and Special Functions consists of a broad set of mathematicians, both pure and applied. The Group also includes engineers and scientists, students as well as experts. We have around 115 members scattered about in more than 20 countries. Whatever your specialty might be, we welcome your participation in this classical, and yet modern, topic. Our WWW home page is:

<http://math.nist.gov/opsf>

This is a convenient point of entry to all the services provided by the Group. Our Webmaster is Bonita Saunders ([bonita.saunders@nist.gov](mailto:bonita.saunders@nist.gov)).

The Activity Group sponsors OP–SF NET, an electronic newsletter, and SIAM-OPSF (OP–SF Talk), a listserv, as a free public service; membership in SIAM is not required. OP–SF NET is transmitted periodically through a post to OP–SF Talk. The OP–SF Net Editors are Howard Cohl ([howard.cohl@nist.gov](mailto:howard.cohl@nist.gov)) and Kerstin Jordaan ([kerstin.jordaan@up.ac.za](mailto:kerstin.jordaan@up.ac.za)).

Back issues of OP–SF NET can be obtained at the websites:

<https://staff.fnwi.uva.nl/t.h.koornwinder/opsfnet>

<http://math.nist.gov/~DLozier/OPSFnet>

SIAM-OPSF (OP–SF Talk), which was recently moved to a SIAM server, facilitates communication among members and friends of the Activity Group. To subscribe or to see a link the archive of all messages, go to <http://lists.siam.org/mailman/listinfo/siam-OPSF> and follow the instructions under the sub-heading “Subscribing to SIAM-OPSF”. To contribute an item to the discussion, send e-mail to [siam-opsf@siam.org](mailto:siam-opsf@siam.org). The moderators are Bonita Saunders ([bonita.saunders@nist.gov](mailto:bonita.saunders@nist.gov)) and Diego Dominici ([dominicd@newpaltz.edu](mailto:dominicd@newpaltz.edu)).

SIAM has several categories of membership, including low-cost categories for students and residents of developing countries. In addition, there is the possibility of reduced rate membership for the members of several societies with which SIAM has a reciprocity agreement; see <http://www.siam.org/membership/individual/reciprocal.php>. For current information on SIAM and Activity Group membership, contact:

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WWW : <http://www.siam.org>

## Topic #12 ——— OP – SF Net 22.4 ——— July 15, 2015

From: OP–SF Net Editors

Subject: Submitting contributions to OP–SF NET and SIAM–OPSF (OP–SF Talk)

To contribute a news item to OP–SF NET, send e-mail to one of the OP–SF Editors

[howard.cohl@nist.gov](mailto:howard.cohl@nist.gov) or [kerstin.jordaan@up.ac.za](mailto:kerstin.jordaan@up.ac.za).

Contributions to OP–SF NET 22.5 should be sent by September 1, 2015.

OP–SF NET is an electronic newsletter of the SIAM Activity Group on Special Functions and Orthogonal Polynomials. We disseminate your contributions on anything of interest to the special functions and orthogonal polynomials community. This includes announcements of conferences, forthcoming books, new software, electronic archives, research questions, and job openings as well as news about new appointments, promotions, research visitors, awards and prizes. OP–SF Net is transmitted periodically through a post to SIAM–OPSF (OP–SF Talk).

SIAM–OPSF (OP–SF Talk) is a listserv of the SIAM Activity Group on Special Functions and Orthogonal Polynomials, which facilitates communication among members, and friends of the Activity Group. See the previous Topic. To post an item to the listserv, send e-mail to [siam-opsf@siam.org](mailto:siam-opsf@siam.org).

WWW home page of this Activity Group:

<http://math.nist.gov/opsf>

Information on joining SIAM and this activity group: [service@siam.org](mailto:service@siam.org)

The elected Officers of the Activity Group (2014–2016) are:

Walter Van Assche, Chair  
Jeff Geronimo, Vice Chair  
Diego Dominici, Program Director  
Yuan Xu, Secretary

The appointed officers are:

Howard Cohl, OP–SF NET co–editor  
Kerstin Jordaan, OP–SF NET co–editor  
Diego Dominici, OP–SF Talk moderator  
Bonita Saunders, Webmaster and OP–SF Talk moderator

## Thought of the month

“Education is not preparation for life; education is life itself.”

John Dewey