

# OP-SF NET – Volume 27, Number 1 – January 15, 2020

The Electronic News Net of the  
SIAM Activity Group on Orthogonal Polynomials and Special Functions

<http://math.nist.gov/opsf>

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**Calendar of Events:**

**March 16–20, 2020**

IV ORTHONET School

Universidad de la Rioja, Logroño, Spain

<http://euler.us.es/orthonet/orthonet20>

**March 21–22, 2020**

AMS Spring Eastern Sectional Meeting

Special Session on “Quantum Probability, Orthogonal Polynomials, and Special Functions”

Tufts University, Medford, Massachusetts, USA

[http://www.ams.org/meetings/sectional/2275\\_program.html](http://www.ams.org/meetings/sectional/2275_program.html)

**April 27–May 1, 2020**

Workshop on Integrable Systems and Orthogonal Polynomials—Numerical  
and Analytical Perspectives

AIMS South Africa, Muizenberg, Cape Town, South Africa

<https://aims.ac.za/2019/10/23/workshop-integrable-systems-and-orthogonal-polynomials-numerical-and-analytical-perspectives/>

**May 11–15, 2020**

LMS–CMI Research School: *Methods for Random Matrix Theory and Applications*  
University of Reading, Reading, UK  
<https://janivirtanen.wordpress.com/research-school-2020>

**May 18–22, 2020**

Baylor Analysis Fest: *From Operator Theory to Orthogonal Polynomials, Combinatorics, and Number Theory*  
Baylor University, Waco, Texas, USA  
<https://www.baylor.edu/math/conference>

**June 15–24, 2020**

Foundations of Computational Mathematics (FoCM2020)  
Workshop on Approximation Theory, June 18–20  
Organized by Albert Cohen, Peter Binev and Maria Charina  
Workshop on Random Matrices, June 18–20  
Organized by Ionna Dumitriu and Sheehan Olver  
Workshop on Special Functions and Orthogonal Polynomials, June 22–24  
Organized by Ana Loureiro, Francisco Marcellán and Andrei Martínez Finkelshtein  
Simon Fraser University, Vancouver, Canada  
<http://focm-society.org/2020/index.html>

**July 5–11, 2020**

8<sup>th</sup> European Congress of Mathematics (8ECM)  
Mini-symposium on Orthogonal Polynomials and Special Functions  
Organized by Paco Marcellán, Juan J. Moreno–Balcázar and Galina Filipuk,  
Portorož, Slovenia  
<https://www.8ecm.si/minisymposia>

**July 6–10, 2020**

SIAM Annual Meeting, held jointly with CAIMS  
(Canadian Applied and Industrial Mathematics Society)  
Sheraton Centre Toronto Hotel, Toronto, Ontario, Canada  
<https://www.siam.org/Conferences/CM/Main/an20>

**July 7–10, 2020**

Functional Analysis, Approximation Theory and Numerical Analysis (FAATNA)  
Matera, Italy  
<http://web.unibas.it/faatna20/>

**July 13–17, 2020**

33<sup>rd</sup> International Colloquium on Group Theoretical Methods in Physics (Group33)  
Cotonou, Benin  
<http://www.cipma.net/group33-cotonou-benin>

**July 13–18, 2020**

Combinatorics around the  $q$ -Onsager algebra, celebrating the 65<sup>th</sup> birthday of Paul Terwilliger  
Satellite event of the 8<sup>th</sup> European Congress of Mathematics  
which will be held the prior week in Portorož, Slovenia,  
Kranjska Gora, Slovenia  
<https://conferences.famnit.upr.si/indico/event/15/overview>

**August 10–14, 2020**

OPPSFA Summer School 2020

Radboud University, Nijmegen, The Netherlands

<https://www.ru.nl/radboudsummerschool/courses/2020/opsfa-summer-school-2020>

**Topic #1 ——— OP – SF Net 27.1 ——— January 15, 2020**

From: OP–SF Net Editors

Subject: Announcement: SIAG–OPSF election results

Nicole Gawel, Membership Coordinator of SIAM, has informed us of the results of the election for the offices of the SIAM Activity Group “Orthogonal Polynomials and Special Functions”.

Elected Name	Position	E-mail Address
Peter Alan Clarkson	Chair	<a href="mailto:P.A.Clarkson@kent.ac.uk">P.A.Clarkson@kent.ac.uk</a>
Luc Vinet	Vice–Chair	<a href="mailto:luc.vinet@umontreal.ca">luc.vinet@umontreal.ca</a>
Andrei Martínez–Finkelshtein	Program Director	<a href="mailto:andrei@ual.es">andrei@ual.es</a>
Teresa E. Pérez	Secretary	<a href="mailto:tperez@ugr.es">tperez@ugr.es</a>

In the words of Nicole Gawel, “*Congratulations to those elected! Thank you to all candidates for their commitment to SIAG/OPSF, and thank you to all who voted!*”. The term of the elected officers started January 1, 2020 and runs until December 31, 2022.

Many thanks to the service of the outstanding outgoing officers! Walter Van Assche and Yuan Xu served respectively as Chair and Secretary of the activity group from 2014–2019. Andrei Martínez–Finkelshtein served as Vice Chair from 2017–2019 (and will now serve as Program Director from 2020–2022). Sarah Post served as Program director from 2017–2019.

We are deeply indebted and greatly appreciative to Walter Van Assche for all the hard work, advice and support that he has given us over the time we have been OP–SF NET co–editors. In our regular activities, we will miss his light wit, his ever–present sense of humor and his talent of being able to drill directly to the main point of any topic which we faced when we needed assistance. It’s been an honor to have worked with him.

**Topic #2 ——— OP – SF Net 27.1 ——— January 15, 2020**

From: Andrei Martínez–Finkelshtein ([A\\_Martinez–Finkelshtein@baylor.edu](mailto:A_Martinez-Finkelshtein@baylor.edu))

and Fritz Gesztesy ([Fritz\\_Gesztesy@baylor.edu](mailto:Fritz_Gesztesy@baylor.edu))

Subject: Announcement: *Baylor Analysis Fest* at Baylor University, Waco, Texas, USA

The international conference “*Baylor Analysis Fest: From Operator Theory to Orthogonal Polynomials, Combinatorics, and Number Theory*” will take place at Baylor University (Waco, Texas, USA), on May 18–22, 2020. It is organized around topics of Analysis such as Operator and Spectral Theory, Special Functions and Orthogonal Polynomials, and their connections with Combinatorics, Probability Theory, and Number Theory. It will also celebrate Lance L. Littlejohn’s accomplishments in building the Mathematics Department at Baylor University for well over a decade now.

Plenary Speakers:

- George E. Andrews (Pennsylvania State University);

- David Damanik (Rice University);
- Lance L. Littlejohn (Baylor University);
- Ken Ono (University of Virginia);
- Barry Simon (California Institute of Technology).

The local Organizing Committee is comprised of Fritz Gesztesy and Andrei Martínez–Finkelshtein. For further details, visit <https://www.baylor.edu/math/conference>.

## Topic #3 ——— OP – SF Net 27.1 ——— January 15, 2020

Proceedings of the School and Conference held at Będlewo, Poland, September 2–15, 2018.  
From: Galina Filipuk ([filipuk@mimuw.edu.pl](mailto:filipuk@mimuw.edu.pl))

Subject: Announcement: CDDE–2018, Będlewo, Poland Proceedings

We would like to draw your attention to the following proceedings:

### **Będlewo CDDE–2018:** *Complex Differential and Difference Equations*

Proceedings of the School and Conference held at Będlewo, Poland, September 2–15, 2018.  
Edited by Galina Filipuk, Alberto Lastra, Sławomir Michalik, Yoshitsugu Takei and Henryk Żołądek.  
Series: [De Gruyter Proceedings in Mathematics](#).

This collection contains many articles on relevant topics that may interest OPSF members including symmetries of hypergeometric functions, special function solutions used in WKB analysis, Painlevé transcendents and solutions of  $q$ -difference equations.

**Aims and Scope:** With a balanced combination of longer survey articles and shorter, peer-reviewed research-level presentations on the topic of differential and difference equations on the complex domain, this edited volume presents an up-to-date overview of areas such as WKB analysis, summability, resurgence, formal solutions, integrability, and several algebraic aspects of differential and difference equations.

<https://www.degruyter.com/view/product/506612>.

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Topic #4 ——— OP – SF Net 27.1 ——— January 15, 2020

From: Ted Chihara ([chihara@pnw.edu](mailto:chihara@pnw.edu)), Werner Scheinhardt ([w.r.w.scheinhardt@utwente.nl](mailto:w.r.w.scheinhardt@utwente.nl)),  
Frank Coolen ([frank.coolen@durham.ac.uk](mailto:frank.coolen@durham.ac.uk)), Lance L. Littlejohn ([Lance\\_Littlejohn@baylor.edu](mailto:Lance_Littlejohn@baylor.edu))  
and Paco Marcellán ([pacomarc@ing.uc3m.es](mailto:pacomarc@ing.uc3m.es))  
Subject: Some remembrances (five) of Erik A. van Doorn

**Some remembrances of  
Erik van Doorn  
(August 12, 1949—November 1, 2019)**

**by Chihara, Scheinhardt, Coolen, Littlejohn and Marcellán**

An obituary of Erik van Doorn appeared in OP–SF Net 26.6, published on November 15, 2019. Below are remembrances of Erik from five of his colleagues and friends, Ted Chihara, Werner Scheinhardt, Frank Coolen, Lance L. Littlejohn and Paco Marcellán.

\* \* \*

**Ted Chihara**, Purdue University Northwest, Hammond, Indiana, USA.

My first contact with Erik occurred in the form of a letter from him dated October 1982. He had sent me a copy of his thesis written in 1979 at Twente University of Technology. This thesis, titled [*“Stochastic Monotonicity of Birth-Death Processes”*, Advances in Applied Probability, 12:1, 59–80, 1980], was subsequently published in a “polished” form as Vol. 4 in the Springer–Verlag Lecture Notes in Statistics [*“Stochastic monotonicity and queueing applications of birth-death processes”*, Lecture Notes in Statistics, 4, Springer–Verlag, New York–Berlin, vi+118 pp., 1981]. In his accompanying letter, he said that at the time he wrote his thesis, he was not aware of the literature on orthogonal polynomials apart from the classic books. And he went on to say that at the time, he did not think of “orthogonal polynomials” as a key phrase. He added that he later read a paper by Jesus Dehesa which had both “orthogonal polynomials” and “birth–death processes” in its title and through it discovered my book. I first met Erik at the conference “Constructive Function Theory–86” held in Edmonton, Alberta in July of 1986. Interestingly, Erik presented a paper on the zeros of orthogonal polynomials whereas I read a paper on the spectra of certain birth–death processes.

As we all know, Erik continued with his interest in OP and as well as with his work on birth–death processes and my next contact with him took place at the OPSFA 3 meeting in Erice, Sicily in 1990 where I also met his charming wife, Janny. Erik’s paper at this conference was on a class of generalized orthogonal polynomials.

One chapter of his thesis was devoted to queueing processes where potential customers are discouraged by queue length. Here and in a later paper [*“The transient state probabilities for a queueing model where potential customers are discouraged by queue length”*, Journal of Applied Probability, 18:2, 499–506, 1981], he succeeded in finding explicitly the spectral function of the process so he has the orthogonality relations for the corresponding orthogonal polynomials, the van Doorn polynomials. His results suggested a characterization problem which led Mourad Ismail and me to a generalization [*“Orthogonal polynomials suggested by a queueing model”*, Advances in Applied Mathematics 3:4, 441–462, 1982] of the van Doorn polynomials. Thus, Erik’s work has proven fruitful to progress in the study of orthogonal polynomials as well as providing much needed further applications. I am grateful that his work has led me to write a few papers on birth–death processes so I can say I have done a little applied mathematics.

\* \* \*

**Werner Scheinhardt**, University of Twente, Enschede, The Netherlands.

Erik had an interest in birth–death processes and random walks due to his passion for orthogonal polynomials, as discussed by Ted Chihara, but he also worked on other topics in stochastic processes, such as convergence of Markov chains and quasi–stationary distributions, to be addressed by Frank Coolen (husband of the late Pauline Schrijner, who worked with Erik on these

topics). It was doing my graduation project on convergence of Markov chains under Erik's encouraging and supportive guidance that made me decide to pursue a scientific career.

Another area to which Erik contributed was stochastic fluid queues, generalizing the foundational Anick–Mitra–Sondhi model, first allowing the modulating process to be any finite birth–death process [*“A fluid reservoir regulated by a birth-death process”*, Erik A. van Doorn, A. A. Jagers and J. S. J. de Wit, *Communications in Statistics. Stochastic Models*, 4:3, 457–472, 1988], and later allowing it to have infinitely many states, and also to let its evolution be influenced by the queue content. The latter results were part of the PhD project I did under Erik's supervision, which resulted in the thesis [*“Markov-modulated and feedback fluid queues”*, Willem Richard Werner Scheinhardt, University of Twente, 155 pp., 1998]. I am thankful for the memories of working with Erik and others on these topics, and of joining him (and Janny!) to visit several Matrix–Analytic Methods conferences. Only later it was recognized in this community that fluid queues can be analyzed in greater generality using matrix–geometric techniques; however much credit for the early developments is due to Erik.

By no means were Erik's interests limited to the areas mentioned above. To mention two of his 'ad hoc', but nevertheless influential, papers: together with Jagers, he wrote [*“On the continued Erlang loss function”*, with Jagers, A. A., *Operations Research Letters*, 5:1, 43–46, 1986], in which conjectured and unknown properties were proven for the so well-known and widely applied formula for the 'blocking probability' in finite-server systems without queueing. And, with Regterschot, he generalized the well-known PASTA property of queueing theory (acronym for Poisson Arrivals See Time Averages) to the context of a random environment [*“Conditional PASTA”*, Erik A. van Doorn and G. J. K. Regterschot, *Operations Research Letters*, 7:4, 229–232, 1988].

Erik van Doorn was a versatile mathematician, and I feel privileged that he was the one who first taught me probability, stochastic processes and queueing, then taught me how to do research, and finally helped me find my own way, while always being available for good advice when needed. He has always been a great support and will continue to be an inspiring example of how research can be done in a meaningful and pleasant way.

\*\*\*

**Frank Coolen**, Durham University, Durham, United Kingdom.

Erik published the paper [*“Quasi-Stationary Distributions and Convergence to Quasi-Stationarity of Birth-Death Processes”*, *Advances in Applied Probability*, 23:4, 683–700, 1991]. In this highly cited paper, he presented three key results: The set of quasi-stationary distributions for birth–death processes, the quasi-limiting distribution for the state of such processes, and the rate of convergence of the transition probabilities of the process conditioned on non-absorption. This work considered processes in continuous time. Together with his PhD student Pauline Schrijner (after our marriage, Coolen–Schrijner), Erik also developed the corresponding theory for discrete-time processes, resulting in Pauline's PhD thesis [*“Quasi-Stationarity of Discrete-Time Markov Chains”*, The University of Twente, 105 pp., 1995], and several related journal publications. The collaboration between Erik and Pauline continued after Pauline had moved to Durham University, leading to several research visits by Erik to North–East England, resulting in further scientific outputs and many pleasant trips, during which Erik provided us with much information about the area. A highlight was a visit to Chester Fort Museum on Hadrian's Wall, where stones contain the first reference to the Tuihanti, the Latin name for the people from Twente.

The collaboration with Pauline also resulted in a further much-cited paper [*“The Deviation Matrix of a Continuous-Time Markov Chain”*, Pauline Coolen–Schrijner and Erik A. van Doorn, *Probability in the Engineering and Informational Sciences*, 16:3, 361–366, 2002]. The many contributions by

Erik on quasi-stationarity, together with related work by many other researchers, partly built on the foundations laid by Erik, are described in an overview paper written together with his regular collaborator and friend Phil Pollett [*“Quasi-Stationary Distributions for Discrete-State Models”*, Erik A. van Doorn and Philip K. Pollett, *European Journal of Operational Research*, **230**:1, 1-14, 2013]. While I have never personally collaborated with Erik, we regularly discussed topics of joint interest, in particular during his visits to Durham, which also continued after Pauline’s untimely death in 2008.

Erik had a wide interest in topics in Probability and related fields, and often provided useful input into discussions on topics which were quite far from his own research. A particularly nice memory I have of Erik is his explanation to us, when the mathematical contents of a research project with Pauline had been completed, that ‘now the really nice work’ started, namely the writing of the paper. Erik enjoyed this and explained that he first carefully thought about the overall structure of the paper. This was followed by detailed consideration of the order and contents of the paragraphs, then the sentences and, finally, the words. He would then leave the paper for a few days, return to it for some final tweaking, after which he asked Pauline if she thought the paper was ready for submission to a journal, which of course it was (and had been for some time in the eyes of most!).

Erik gave similar care to his presentations, which one could largely follow even without being a topic expert. Such care of presenting his scientific work, together with the quality of his contributions, makes Erik an example to us all. Far beyond this, Erik was a caring person with a great sense of humor, who will be missed by all who had the pleasure to know him.

\* \* \*

**Lance L. Littlejohn**, Baylor University, Waco, Texas, USA.

Like Ted Chihara, I first met Erik at the Constructive Function Meeting in Edmonton in 1986. He was staying in a dorm room next to mine and we happened to meet one evening outside our rooms. This was the start of a wonderful friendship with Erik, and a little later, with his delightful wife, Janny. For me, personally, it was great fun every two years to meet up with Erik, Janny and Ted at the OPSFA meetings.

In the fall of 1990, I started a year-long sabbatical leave at the University of Wales in Cardiff when I was invited to be a member of the committee for the Ph.D. defense of Roelof Koekoek at Delft University. After that trip, the three Littlejohns visited the van Doorns in Enschede. Oh, what a visit we had! Erik and Janny had planned a wonderful visit for us; among several other places we went, we visited a wooden shoe factory (I still wear mine when working in the garden!) and our four-year old son had a memorable visit with Sinterklaas. Erik had recently returned from a mathematical trip to East Germany. He brought his young son Tim a real cross bow. If I am recalling this incident correctly, an errant arrow knocked out one of their windows!

Erik and Janny visited us in Logan, Utah and Waco, Texas. So much fun each time for the four of us! Janny is an extraordinarily gifted oil and water painter and a talented potter and sculptor as well. Wendy and I recently counted the number of art pieces that Janny has made for us over the years: eight. We proudly have each of them displayed throughout our home. She made a ninth piece of pottery recently for our new granddaughter.

Similar to Frank Coolen’s comment, Erik and I never collaborated on any research projects. We both viewed the subject of orthogonal polynomials more as a hobby as we pursued research in different areas which had orthogonal polynomials as applications to our work.

We have lost both a wonderful friend and an accomplished mathematician. I counted Erik as one of my truly close and dear friends.

**Paco Marcellán**, Universidad Carlos III de Madrid, Madrid, Spain.

On October 31 of last year, Lance L. Littlejohn sent me the following email “*Today, I received a very sad phone call from Janny van Doorn who told me that her husband, Erik, was killed riding his bicycle yesterday. Janny wanted me to inform the orthogonal polynomials group that they both knew and loved for many, many years*”. I was saddened by the bad news during this Fall semester (the death of Martin Muldoon and Dick Askey, among those who were closest to me.)

I met Erik in Segovia, Spain, in September 1986 on the occasion of the OPSFA meeting, the second one in the series which started in Bar–Le–Duc, France, in 1984. I was one of the organizers of that meeting with Manuel Alfaro, Jesús S. Dehesa, Jaime Vinuesa and José Luis Rubio de Francia (one of the top Spanish mathematicians who sadly died at the age of 38 in 1988). For me it was a great opportunity to meet people from several different countries in Europe, and as well from the former Soviet Union and from America (among them a young Lance L. Littlejohn who became a very good friend of mine as well as Erik’s). In a period when mathematical connections were mainly supported by mail and using the journals as a basic way, scientific meetings were the right place to show the face of people whose work was related to my scientific interests. I think it was a nice and successful meeting where we shared not only mathematics, but had the honor to stay in one of the most beautiful cities in Spain.

The study of stochastic process has been the main research interest of Erik’s. His doctoral dissertation on stochastic monotonicity of Birth and Death processes, defended in Twente University in 1980 under the supervision of J. A. H. de Smit (his name is missed in the Mathematics Genealogy project) was the starting point of a very successful career covering topics including birth–death processes and random walks with applications to orthogonal polynomials. He was also involved in many other areas of stochastic processes, including some relevant contributions in quasi-stationary distributions, convergence of Markov chains and stochastic fluid queues.

Erik was a real gentleman as well as an excellent mathematician. I asked his advice about a joint paper with Gerardo Pérez, a student of mine, on *The moments of the  $M/M/s$  queue length process*. He suggested many improvements to us for our presentation and also considerations for the right journal (Queueing Systems: Theory and Applications) in order to submit it. Indeed, it was published therein in 2003.

His work on orthogonal polynomials combines a good expertise on classical topics (zeros, spectral properties, asymptotics) together with motivation from stochastic processes. I would like to point out that the last paper by Erik available in MathSciNet is a joint work with R. Szwarc, “*On a property of random walk polynomials involving Christoffel functions*”, J. Math. Anal. Appl, **477**:1 (2019), pp. 85–103. By using Christoffel functions for a type of [orthogonal polynomials](#) known as a random walk *polynomials*, they proved the equivalence of asymptotic aperiodicity and the strong ratio limit property for a normalized birth–death process under mild regularity conditions.

Last but not least, as a universal constant, in all pictures when I am appear with Erik, also appears Ted Chihara and Janny, Erik’s wonderful wife. As a sample, enclosed I show a picture where people of our community of orthogonal polynomials and special functions can be “*easily*” identified (around 1990). The exercise I suggest to the readers of this remembrance is where the picnic took place. (The solution is given in the next page).



Photograph taken at the Third International Symposium on Orthogonal Polynomials and their Applications ([OPSFA-3](#)) in Erice, Sicily, Italy, May 31–June 9, 1990.

I to r: Ted Chihara, Janny van Doorn (Erik's wife), Erik van Doorn,  
Manuel Alfaro (his back is to the photographer),  
Walter van Assche, Jet Wimp, Walter Gautschi, Ed Saff and Paco Marcellán.

Topic #5 ——— OP – SF Net 27.1 ——— January 15, 2020

From: Mourad E. H. Ismail ([mourad.eh.ismail@gmail.com](mailto:mourad.eh.ismail@gmail.com))

Subject: Report by **Mourad E. H. Ismail**: Askey family memorial service

The Askey family held a memorial service for Dick Askey on November 9, 2019. It was held at the Chapel in the Oakwood Village Prairie Ridge in Madison, Wisconsin. The service was conducted by Rev. Eldonna Hazen and Chaplain Wayne Shannon. Opening speeches were given by Dick's kids, Suzanne and Jim followed by Ranjan Roy who also read words from George Andrews. Hung-Hsi Wu from Berkeley and Mourad Ismail also gave speeches. David Foss, Suzanne's husband read a moving "How Do I Love Thee". Several professors from the University of Wisconsin, Departments of Mathematics and Computer Science attended the ceremony. The speeches were very emotional and touched on many aspects of Dick's life and his dedication to his family, mathematics and mathematics education.

After the service, refreshments were served and we had a chance to chat. We all remembered Dick's kindness, mentoring, caring, and impact on many people's lives.

We miss you Dick, and you will always be in our hearts and minds.

From: Howard S. Cohl ([howard.cohl@nist.gov](mailto:howard.cohl@nist.gov)) [and Suzanne Askey ([askey@chorus.net](mailto:askey@chorus.net))]  
 Subject: Report by **Howard S. Cohl**: *Askey Liber Amicorum*, a Friendship Book for Dick Askey

Upon learning of Dick Askey's illness and his admittance into Hospice Care in Madison, Wisconsin on September 4, 2019, Howard Cohl contacted Mourad Ismail inquiring whether we might prepare a *Liber Amicorum*, a Friendship Book, for Dick Askey. Mourad thought that this was a great idea and indicated that Howard should take the lead in its preparation. After all, we both would be attending the Fall AMS Sectional Meeting at the University of Wisconsin, Madison on Saturday and Sunday September 14–15, 2019 to attend a Special Session on *Special Functions and Orthogonal Polynomials* organized by Paul Terwilliger and Sarah Post. We decided that if all worked out, we could print and present the Friendship Book to Dick on one of the days of the meeting in Madison. So began an intense week and a half long effort to contact, communicate and coordinate with Dick's family, the OPSF community, and all who may have known and worked with Dick, to inquire whether they might contribute a short tribute or an interesting piece of mathematics to be included in the *Liber Amicorum*—which might also be read out loud on the day we would present the Friendship Book to Dick.

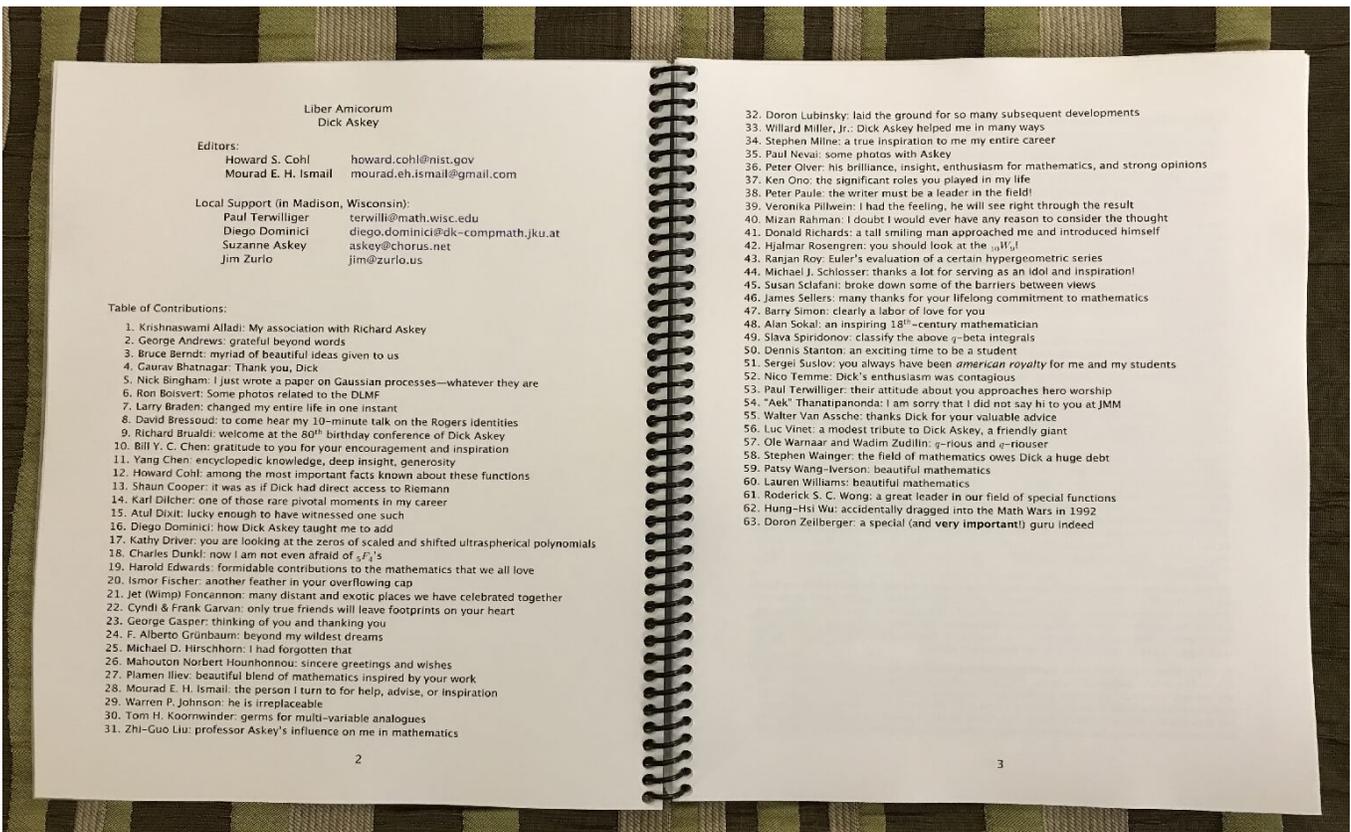
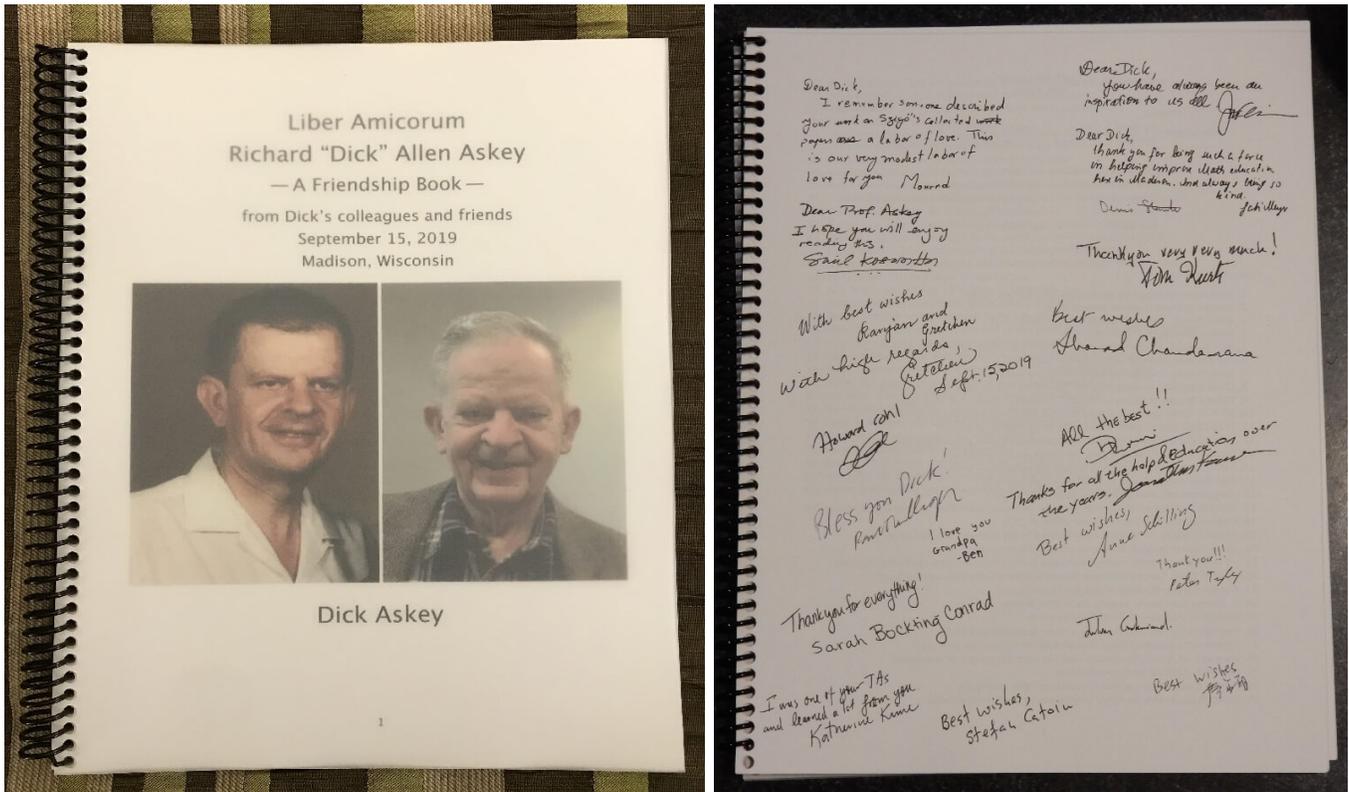


Photo of original Table of Contributions for the first edition *Liber Amicorum*, a Friendship Book for Dick Askey. Photo taken by Howard Cohl.

On Saturday, September 14, after the last talk of the session in the afternoon, Howard Cohl and Mourad Ismail met Jim Zurlo, Dick's son, on the street at the University of Wisconsin. Jim had offered to drive Howard and Mourad to a FedEx Office Print & Ship Center near the University of Wisconsin in order to obtain a printed copy of the *Liber Amicorum*. The intention was to present the Friendship Book to Dick on the following day. After a discussion about the requested parameters of the printing, Jim had to get back to his family, and Howard and Mourad had dinner near

the FedEx shop while waiting for the Friendship Book to be printed. The printing took about an hour. We chose the best possible paper which was thick enough that the photographs would not bleed through the opposite pages, and printed the book double sided and used a spiral binding. The printed first edition Liber Amicorum ended up being quite beautiful. It contained 63 contributions, with 62 color photographs and was 106 pages long.



Photos of the cover and signature page for the first edition Liber Amicorum, a Friendship Book for Dick Askey. This original book printed on September 14, 2019, was presented to Dick in Madison, Wisconsin on the morning of Sunday September 15, 2019. On that same morning, the Friendship Book was signed by the following individuals. First column: Mourad Ismail, unknown signature, Ranjan and Gretchen Roy, Howard Cohl, Paul Terwilliger, Ben Hinkel (Suzanne's son and Dick's grandson), Sarah Bocking-Conrad, Katherine Kime, Stefan Catoiu. Second column: Jin-Yi Cai, unknown signature, Dennis Stanton, Tom Kurtz, Sharad Chandarana, Diego Dominci, Jonathan Kane, Anne Schilling, Peter Tingley, Julien Gaboriaud, Hanmeng (Harmony) Zhan. Cover page photo taken by Howard Cohl; Signature page photo taken by Jim Zurlo, Dick's son.

On Sunday, September 15, after the last talk of the morning session, two groups of mathematicians boarded two separate cars to drive to a chapel at Oakwood Village Prairie Ridge, about 10 miles northeast of downtown Madison. The first car was driven by Thomas Kurtz, Emeritus Professor at the University of Wisconsin. He drove Mourad Ismail and Dennis Stanton to the chapel. The second car was driven by Peter Tingley, Loyola University Chicago, who drove Howard Cohl, Diego Dominici, and Anne Schilling to the chapel. Ranjen Roy and his wife drove to the chapel in a separate car from Beloit, Wisconsin, where they live.

After arriving at the Chapel, Dick had not arrived yet, but his family was there as well as several colleagues from the University and elsewhere. Most of the people who were present (there were about 30 people) signed an empty page in the front of the Friendship Book. Dick was sleeping, but was about to wake up and before long he arrived in a wheel chair with a nurse. He was positioned at the front of a circle of chairs with his wife Liz next to him, as well as his son Jim and daughter Suzanne.



l to r: Mourad Ismail, Jim Zurlo (Dick's son), Liz Askey (Dick's wife), Dick Askey, and Suzanne Askey (Dick's daughter). Photo taken by Howard Cohl.



Dick Askey and Suzanne Askey browsing the Liber Amicorum. Photo taken by Howard Cohl.

The whole emotional ceremony lasted approximately 45 minutes. When Dick was ready to start, Suzanne made some short comments. Suzanne pretty much ran the whole show. Howard stood up, gave a short description of the contents of the Friendship Book, and presented it to him. Dick, who wasn't able to move quickly, leafed through the pages and examined the front cover and the table of contents. Suzanne leafed through the book for Dick and showed him some of the many historical photos of Dick and his colleagues and friends, which were contained within the book.

Howard then read out loud the full list of people who had provided written contributions to the Friendship Book. (After Howard’s listing of contributors, Mourad pointed out that Howard had left out mentioning his own contribution. Howard located and then affirmed its existence, which brought out laughter in the room.) Dick seemed to be quite emotional during these moments. Afterwards, Mourad gave a short speech about the idea of the Friendship book, describing all the people who responded to our request for contributions, and that they all love Dick, and especially Mourad himself who thanked Dick for all the difference Dick had made in so many people’s lives. The whole while Suzanne was visibly comforting Dick.

Mourad then introduced Diego Dominici who presented the OPSFA Lifetime Achievement Award to Dick. (This had previously been shown at the OPSFA–15 meeting in Hagenberg, Austria.) Sev-



Dick Askey’s OPSFA Lifetime Achievement Award, presented to Dick Askey by Diego Dominici in Madison, Wisconsin on Sunday September 15, 2019.

Photo taken by Peter Paule.

eral people spoke and made some statements about Dick and to Dick, and asked questions of Dick. Dick said a few words about the OPSFA meetings. Both Mourad and Dick talked about the history of the OPSFA meetings and in particular, OPSFA–1, Bar-le-Duc, France, 1984. Dick mentioned that there was fireworks during that meeting! Dick also referred to his Bronze bust of Srinivasa Ramanujan, one of ten copies cast in 1983, which has been recently donated by Dick’s family to the Department of Mathematics at the University of Wisconsin—and will be installed in their 9<sup>th</sup> floor lounge.

Dick, Suzanne Askey and Jim Zurlo made several extended comments. Suzanne asked Howard to read some contributions from the Friendship book and Howard read the selected contributions of George Andrews, Barry Simon, and Doron Zeilberger. Dick also reminisced about the fact that he got the idea for the Askey–scheme at an Oberwolfach meeting in 1977 on “*Combinatorics and Special Functions*”. He mentioned that Michael Hoare, in connection with his lecture, distributed

copies of a sheet which contained in graphical way, a part of the present Askey scheme, and which was received very enthusiastically by the audience.

Note that a second edition of the *Liber Amicorum* is currently being finalized which will contain at least 84 contributions about Dick and 95 color photographs of Dick.

*Note added in Proof (by Suzanne Askey).*—Dick lived another 3 weeks and up until the last couple days, every day I read to him from the *Liber Amicorum*. When I was reading to him he perked up, opened his eyes and focused more than he was able to most of the rest of the time. He was very interested and glad to be hearing all that had been written by mathematicians around the world. He was able to hear all of the contributions before he died. I can't put into words how grateful I am that his last weeks were full of the appreciation, respect, and love from all those who contributed to the *Liber Amicorum*, and the work put into making it by Howard and the upfront work by Mourad in starting the emails of appreciation for Dick pouring into my email. With much gratitude, Suzanne Askey (Dick's daughter).

## Topic #7 ——— OP – SF Net 27.1 ——— January 15, 2020

From: Juan J. Moreno–Balcázar ([balcazar@ual.es](mailto:balcazar@ual.es))

Subject: Report by **Juan J. Moreno–Balcázar**: *Two Days on Orthogonal Polynomials*

Report on the 2<sup>nd</sup> Workshop “Two Days on Orthogonal Polynomials & Special Functions”  
Dates: November 21–22, Almería, Spain.

In 1994 Andrei Martínez–Finkelshtein arrived to the Universidad de Almería and he created the research group *Teoría de aproximación y polinomios ortogonales* (TAPO–Approximation Theory and Orthogonal Polynomials) with members of the Universidad de Almería and Universidad de Granada. To celebrate the 25<sup>th</sup> Anniversary of our research group, we decided to carry out this workshop, that gave continuity to a previous one held in Granada last year. The topics tackled, according to the title of the workshop, were orthogonal polynomials and special functions.

The workshop was very intense with 19 plenary talks given by speakers belonging to the research groups on orthogonal polynomials from Spain and a poster session. It was inaugurated by the Vice Chancellor of Research and Innovation, Diego Luis Valera, Universidad de Almería.

On Thursday morning, the talks were given by Paco Marcellán (Universidad Carlos III de Madrid), Andrei Martínez–Finkelshtein (Baylor University and Universidad de Almería), María José Cantero (Universidad de Zaragoza), Amparo Gil (Universidad de Cantabria), José Luis López (Universidad Pública de Navarra) and Ester Pérez (Universidad de Zaragoza).

The talks on the Friday sessions were given by Jesús Sánchez–Dehesa (Universidad de Granada), Antonio Durán (Universidad de Sevilla), Renato Álvarez (Universidad de Sevilla), Ramón Orive (Universidad de La Laguna), Héctor Pijeira (Universidad Carlos III de Madrid), Guillermo López (Universidad Carlos III de Madrid), Óscar Ciaurri (Universidad de la Rioja), Miguel Piñar (Universidad de Granada), Iván Area (Universidad de Vigo), Juan Luis Varona (Universidad de la Rioja), Manuel Mañas (Universidad Complutense) and David Gómez–Ullate (Universidad de Cádiz).

A special invited talk was given by Gracia Castro (Universidad de Almería and INVISION firm), connecting orthogonal polynomials with medical problems in ophthalmology. It was a nice talk to finish an intense morning.

All the talks were very interesting and motivated the scientific discussion among the participants in the workshop. As a curiosity, the talks were in Spanish but in many cases the slides of the presentations were in English.

The social programme took place on Thursday afternoon. We visited the *Shelters of the Spanish Civil War* which were constructed by the architect Guillermo Langle Rubio (1895–1981) to safeguard the civil population from air strikes during the Spanish Civil War (1936–1939). Later, we could enjoy some places around the city where very famous movies were filmed such as *Patton* (1970) and *Indiana Jones and the Last Crusade* (1989). Finally, the dinner of the meeting was in a well-known restaurant near the marina.

We hope that we can celebrate other anniversaries and join people in Almería once again.



Official photo of the 2<sup>nd</sup> Workshop “Two Days on Orthogonal Polynomials and Special Functions”, November 21–22, 2019, Almería, Spain.

Topic #8 ——— OP – SF Net 27.1 ——— January 15, 2020

From: OP–SF Net Editors  
Subject: Preprints in arXiv.org

The following preprints related to the fields of orthogonal polynomials and special functions were posted or cross-listed to one of the subcategories of arXiv.org during November and December 2019. This list has been separated into two categories.

### OP–SF Net Subscriber E-Prints

<http://arxiv.org/abs/1911.01423>

Two Definite Integrals That Are Definitely (and Surprisingly!) Equal  
Shalosh B. Ekhad, Doron Zeilberger, Wadim Zudilin

<http://arxiv.org/abs/1911.01744>

Solutions for the Lévy–Leblond or parabolic Dirac equation and its generalizations  
Sijia Bao, Denis Constaes, Hendrik De Bie, Teppo Mertens

<http://arxiv.org/abs/1911.02612>

Periodic Jacobi Matrices on Trees  
Nir Avni, Jonathan Breuer, Barry Simon

<http://arxiv.org/abs/1911.03707>

Where do the maximum absolute  $q$ -series coefficients of  $(1 - q)(1 - q^2)(1 - q^3) \dots (1 - q^{n-1})(1 - q^n)$  occur?

Alexander Berkovich, Ali K. Uncu

<http://arxiv.org/abs/1911.04849>

Further equidistribution of set-valued statistics on permutations  
Jianxi Mao, Jiang Zeng

<http://arxiv.org/abs/1911.04905>

Complex asymptotics in lambda for the Gegenbauer functions  $C_\lambda^\alpha(z)$  and  $D_\lambda^\alpha(z)$  with  $z \in (-1, 1)$   
Loyal Durand

<http://arxiv.org/abs/1911.05854>

On the Riemann–Hilbert problem for a  $q$ -difference Painlevé equation  
Nalini Joshi, Pieter Roffelsen

<http://arxiv.org/abs/1911.06375>

The Boundedness of the Ornstein–Uhlenbeck semigroup on variable Lebesgue spaces with respect to the Gaussian measure  
Jorge Moreno, Ebner Pineda, Wilfredo Urbina

<http://arxiv.org/abs/1911.08314>

Howe duality and algebras of the Askey–Wilson type: an overview  
Julien Gaboriaud, Luc Vinet, Stéphane Vinet

<http://arxiv.org/abs/1911.08732>

A crystal on decreasing factorizations in the 0-Hecke monoid  
Jennifer Morse, Jianping Pan, Wencin Poh, Anne Schilling

<http://arxiv.org/abs/1911.10533>

Asymptotics of Polynomials Orthogonal on a Cross with a Jacobi-type Weight  
Ahmad Barhoumi, Maxim L. Yattselev

<http://arxiv.org/abs/1911.10602>

Corrigendum on the proof of completeness for exceptional Hermite polynomials  
David Gomez–Ullate, Yves Grandati, Robert Milson

<http://arxiv.org/abs/1911.11280>

Zero sets, entropy, and pointwise asymptotics of orthogonal polynomials  
Roman Bessonov, Sergey Denisov

<http://arxiv.org/abs/1911.11316>

Harmonic analysis for rank-1 Randomised Horn Problems  
Jiyuan Zhang, Mario Kieburg, Peter J. Forrester

<http://arxiv.org/abs/1911.11925>

An Algebraic Geometric Foundation for a Classification of Superintegrable Systems in Arbitrary Dimension  
Jonathan Kress, Konrad Schöbel, Andreas Vollmer

<http://arxiv.org/abs/1911.12195>

Minimal energy point systems on the unit circle and the real line  
Marcell Gaál, Béla Nagy, Zsuzsanna Nagy-Csiha, Szilárd Révész

<http://arxiv.org/abs/1911.12746>

Discrete-Continuous Jacobi-Sobolev Spaces and Fourier Series  
Abel Díaz-González, Francisco Marcellán-Español, Héctor Pijeira-Cabrera, Wilfredo Urbina-Romero

<http://arxiv.org/abs/1912.00353>

Quasi-orthogonality and zeros of some  ${}_2\varphi_2$  and  ${}_3\varphi_2$  polynomials  
P. P. Kar, K. Jordaan, P. Gochhayat, M. K. Nangho

<http://arxiv.org/abs/1912.00765>

$q$ -Supercongruences modulo the fourth power of a cyclotomic polynomial via creative microscoping  
Victor J. W. Guo

<http://arxiv.org/abs/1912.01199>

On Hurwitz zeta function and Lommel functions  
Atul Dixit, Rahul Kumar

<http://arxiv.org/abs/1912.01475>

A new family of orthogonal polynomials in three variables  
Rabia Aktaş, Iván Area, Esra Göldoğan

<http://arxiv.org/abs/1912.02142>

Critical Behavior of Non-Intersecting Brownian Motions  
Tom Claeys, Thorsten Neuschel, Martin Venker

<http://arxiv.org/abs/1912.02282>

Bound states and the potential parameter spectrum  
A. D. Alhaidari, H. Bahlouli

<http://arxiv.org/abs/1912.02377>

Analytical Approximations to the Dynamics of Nonlinear Level Crossing Models  
Chon-Fai Kam, Yang Chen

<http://arxiv.org/abs/1912.02957>

Shift-invariance for vertex models and polymers  
Alexei Borodin, Vadim Gorin, Michael Wheeler

<http://arxiv.org/abs/1912.03390>

Compact formulas for Macdonald polynomials and quasisymmetric Macdonald polynomials  
Sylvie Corteel, Jim Haglund, Olya Mandelshtam, Sarah Mason, Lauren Williams

<http://arxiv.org/abs/1912.03674>

Inversion sequences avoiding pairs of patterns  
Chunyan Yan, Zhicong Lin

<http://arxiv.org/abs/1912.03680>

Analytic properties of sextet polynomials of hexagonal systems  
Guanru Li, Lily Li Liu, Yi Wang

<http://arxiv.org/abs/1912.03689>

Proofs of some partition identities conjectured by Kanade and Russell  
Hjalmar Rosengren

<http://arxiv.org/abs/1912.04599>

Global fluctuations for Multiple Orthogonal Polynomial Ensembles  
Maurice Duits, Benjamin Fahs, Rostyslav Kozhan

<http://arxiv.org/abs/1912.04605>

On algebraic Stein operators for Gaussian polynomials  
Ehsan Azmoodeh, Dario Gasbarra, Robert E. Gaunt

<http://arxiv.org/abs/1912.06245>

Connectivity concerning the last two subconstituents of a  $Q$ -polynomial distance-regular graph  
Sebastian M. Cioabă, Jack H. Koolen, Paul Terwilliger

<http://arxiv.org/abs/1912.06345>

The Irrationality Measure of  $\pi$  is at most 7.103205334137...  
Doron Zeilberger, Wadim Zudilin

<http://arxiv.org/abs/1912.06488>

Polynomial Representations of the Lie Superalgebra  $osp(1|2n)$   
A.K. Bisbo, H. De Bie, J. Van der Jeugt

<http://arxiv.org/abs/1912.06651>

Some remarks on generalized Fibonacci and Lucas polynomials  
Johann Cigler

<http://arxiv.org/abs/1912.06829>

The method of creative microscoping  
Wadim Zudilin

<http://arxiv.org/abs/1912.07151>

Constructing high order spherical designs as a union of two of lower order  
Mozhgan Mohammadpour, Shayne Waldron

<http://arxiv.org/abs/1912.07533>

Orthogonal structure and orthogonal series in and on a double cone or a hyperboloid  
Yuan Xu

<http://arxiv.org/abs/1912.07597>

Lectures on exceptional orthogonal polynomials and rational solutions to Painlevé equations  
David Gómez-Ullate, Robert Milson

<http://arxiv.org/abs/1912.08070>

Proof of a supercongruence conjectured by Sun through a  $q$ -microscope  
Victor J. W. Guo

<http://arxiv.org/abs/1912.08658>

Entanglement entropy of two disjoint intervals separated by one spin in an XX quantum spin chain  
L. Brightmore, G. P. Geher, A. R. Its, V. E. Korepin, F. Mezzadri, M. Y. Mo, J. A. Virtanen

<http://arxiv.org/abs/1912.08959>

Discrete Painlevé Equations  
Nalini Joshi

<http://arxiv.org/abs/1912.09037>

Universality near the gradient catastrophe point in the semiclassical sine-Gordon equation  
Bing-Ying Lu, Peter D. Miller

<http://arxiv.org/abs/1912.09377>

Continuity of Weighted Operators in  $A_p$  Weights and Steklov Problem for Orthogonal Polynomials  
Michel Alexis, Alexander Aptekarev, Sergey Denisov

<http://arxiv.org/abs/1912.09698>

Levin methods for highly oscillatory integrals with singularities  
Yinkun Wang, Shuhuang Xiang

<http://arxiv.org/abs/1912.10259>

On Christol's conjecture  
Y. Abdelaziz, C. Koutschan, J-M. Maillard

<http://arxiv.org/abs/1912.10381>

Automatic Discovery of Irrationality Proofs and Irrationality Measures  
Doron Zeilberger, Wadim Zudilin

<http://arxiv.org/abs/1912.10728>

On the Sheffer-type polynomials related to the Mittag-Leffler functions: applications to fractional evolution equations  
K. Górska, A. Horzela, K. A. Penson, G. Dattoli

<http://arxiv.org/abs/1912.11266>

Beyond the beta integral method: transformation formulas for hypergeometric functions via Meijer's  $G$  function  
D. B. Karp, E. G. Prilepkina

<http://arxiv.org/abs/1912.11514>

Superconformal indices, Seiberg dualities and special functions  
Vyacheslav P. Spiridonov

<http://arxiv.org/abs/1912.11571>

The rational Heun operator and Wilson biorthogonal functions  
Satoshi Tsujimoto, Luc Vinet, Alexei Zhedanov

<http://arxiv.org/abs/1912.12019>

Isospectral flows related to Frobenius-Stickelberger-Thiele polynomials  
Xiang-Ke Chang, Xing-Biao Hu, Jacek Szmigielski, Alexei Zhedanov

<http://arxiv.org/abs/1912.12711>

Positive intertwiners for Bessel functions of type B  
Margit Rösler, Michael Voit

<http://arxiv.org/abs/1912.12971>

Introduction to the theory of elliptic hypergeometric integrals  
V. P. Spiridonov

<http://arxiv.org/abs/1912.12974>

Exponentially small expansions related to the parabolic cylinder function  
R. B. Paris

## Other Relevant OP–SF E–Prints

<http://arxiv.org/abs/1911.00186>

New series identities with Cauchy, Stirling, and harmonic numbers, and Laguerre polynomials  
Khristo N. Boyadzhiev

<http://arxiv.org/abs/1911.01029>

Ramanujan sum and Chebotarev densities  
Biao Wang

<http://arxiv.org/abs/1911.01076>

Interpolating the Derivatives of the Gamma Function  
Vassilis G. Papanicolaou

<http://arxiv.org/abs/1911.01087>

Frobenius' theta function and Arakelov invariants in genus three  
Robin de Jong

<http://arxiv.org/abs/1911.01322>

The matching condition for larger size Riemann–Hilbert problems  
Leslie Molag

<http://arxiv.org/abs/1911.01350>

Invariants of models of genus one curves via modular forms and determinantal representations  
Manh Hung Tran

<http://arxiv.org/abs/1911.01514>

On the Form of Solutions of Fuchsian differential Equations with  $n$  regular singular Points  
Albert Huber

<http://arxiv.org/abs/1911.01790>

Proof of two supercongruences by the Wilf–Zeilberger method  
Guo–Shuai Mao

<http://arxiv.org/abs/1911.02004>

A note on Legendre, Hermite, Chebyshev, Laguerre and Gegenbauer wavelets with an application on sbvps arising in real life  
Amit K. Verma, Diksha Tiwari

<http://arxiv.org/abs/1911.02008>

Machine Learning meets Number Theory: The Data Science of Birch–Swinnerton–Dyer  
Laura Alessandretti, Andrea Baronchelli, Yang–Hui He

<http://arxiv.org/abs/1911.02015>

Elliptic functions and flotation  
P. L. Robinson

<http://arxiv.org/abs/1911.03174>

Infinite dimensional systems of particles with interactions given by Dunkl operators  
Andrei Velicu

<http://arxiv.org/abs/1911.03190>

Probabilistic Models for Gram’s Law  
Cătălin Hanga, Christopher Hughes

<http://arxiv.org/abs/1911.03252>

Linear integrable systems on quad–graphs  
Alexander I. Bobenko, Yuri B. Suris

<http://arxiv.org/abs/1911.03942>

Question about integral of product of four Hermite polynomials integrated with squared weight  
Alexander Minakov

<http://arxiv.org/abs/1911.03968>

The fastest series for  $1/\pi$  due to Ramanujan. (A complete proof using Maple)  
Jesús Guillera

<http://arxiv.org/abs/1911.04029>

On a Hilbert Space Reformulation of Riemann Hypothesis  
Boqing Xue

<http://arxiv.org/abs/1911.04141>

$\mathbb{Q}$ –linear dependence of certain Bessel moments  
Yajun Zhou

<http://arxiv.org/abs/1911.04795>

A study on the fixed points of the  $\gamma$  function  
Andrea Frosini, Giulia Palma, Elisa Pergola, Simone Rinaldi

<http://arxiv.org/abs/1911.05456>

New series for powers of  $\pi$  and related congruences  
Zhi–Wei Sun

<http://arxiv.org/abs/1911.05467>

ChebNet: Efficient and Stable Constructions of Deep Neural Networks with Rectified Power Units using Chebyshev Approximations  
Shanshan Tang, Bo Li, Haijun Yu

<http://arxiv.org/abs/1911.05583>

Fast Computation of Orthogonal Systems with a Skew–symmetric Differentiation Matrix  
Arieh Iserles, Marcus Webb

<http://arxiv.org/abs/1911.05688>

On the Relativized Alon Second Eigenvalue Conjecture I: Main Theorems, Examples, and Outline of Proof

Joel Friedman, David Kohler

<http://arxiv.org/abs/1911.05883>

A ratio of many gamma functions and its properties with applications

Feng Qi, Wen-Hui Li, Shu-Bin Yu, Xin-Yu Du, Bai-Ni Guo

<http://arxiv.org/abs/1911.05982>

Stable equilibria for the roots of the symmetric continuous Hahn and Wilson polynomials

J.F. van Diejen

<http://arxiv.org/abs/1911.05984>

Rate of approximation of  $z f'(z)$  by special sums associated with the zeros of the Bessel polynomials

Mikhail A. Komarov

<http://arxiv.org/abs/1911.06037>

Slice Fueter-regular functions

Riccardo Ghiloni

<http://arxiv.org/abs/1911.06060>

A zeta function related to the transition matrix of the discrete-time quantum walk on a graph

Norio Konno, Iwao Sato, Etsuo Segawa

<http://arxiv.org/abs/1911.06115>

Asymptotic formulas for harmonic series in terms of a non-trivial zero on the critical line

Artur Kawalec

<http://arxiv.org/abs/1911.07059>

On Hankel matrices commuting with Jacobi matrices from the Askey scheme

František Štampach, Pavel Šťovíček

<http://arxiv.org/abs/1911.07129>

On the Multiple Zeta Values  $\zeta(\{2\}^k)$

Mario DeFranco

<http://arxiv.org/abs/1911.07604>

Two New Identities Involving the Catalan Numbers: A classical approach

Helmut Prodinger

<http://arxiv.org/abs/1911.07901>

Radii problems for normalized hyper-Bessel function

Evrin Toklu, Osman Kara

<http://arxiv.org/abs/1911.08218>

New explicitly diagonalizable Hankel matrices related to the Stieltjes-Carlitz polynomials

František Štampach, Pavel Šťovíček

<http://arxiv.org/abs/1911.08458>

A note on the number of irrational odd zeta values

Li Lai, Pin Yu

<http://arxiv.org/abs/1911.08767>

Groups, Jacobi functions and rigged Hilbert spaces  
E. Celeghini, M. Gadella, M. A. del Olmo

<http://arxiv.org/abs/1911.08841>

Two properties of the partial theta function  
Vladimir Petrov Kostov

<http://arxiv.org/abs/1911.09139>

Certain hybrid polynomials associated with Sheffer sequences  
Nabiullah Khan, Talha Usman, Mohd Aman

<http://arxiv.org/abs/1911.09486>

Structures de Frobenius forte rigidité et équations hypergéométriques  
Daniel Vargas Montoya

<http://arxiv.org/abs/1911.09497>

Proof of some hypergeometric congruences via the WZ method  
Chen Wang

<http://arxiv.org/abs/1911.09700>

Algebraic solution to constrained bi-criteria decision problem of rating alternatives through pairwise comparisons  
Nikolai Krivulin

<http://arxiv.org/abs/1911.10282>

Titchmarsh–Weyl formula for the spectral density of a class of Jacobi matrices in the critical case  
Serguei Naboko, Sergey Simonov

<http://arxiv.org/abs/1911.10288>

On sequences associated to the invariant theory of rank two simple Lie algebras  
Alin Bostan, Jordan Tirrell, Bruce W. Westbury, Yi Zhang

<http://arxiv.org/abs/1911.10319>

Elementary hypergeometric functions, Heun functions, and moments of MKZ operators  
Ana Maria Acu, Ioan Rasa

<http://arxiv.org/abs/1911.10465>

Meromorphy of local zeta functions in smooth model cases  
Joe Kamimoto, Toshihiro Nose

<http://arxiv.org/abs/1911.10475>

Asymptotic behavior of orthogonal polynomials without the Carleman condition  
Dmitri Yafaev

<http://arxiv.org/abs/1911.10491>

On supercongruences for truncated sums of squares of basic hypergeometric series  
Mohamed El Bachraoui

<http://arxiv.org/abs/1911.10507>

The Christoffel problem by fundamental solution of the Laplace equation  
Qi-Rui Li, Dongrui Wan, Xu-Jia Wang

<http://arxiv.org/abs/1911.10586>

$(G'/G)$ -Expansion Method and Weierstrass Elliptic Function Method Applied to Coupled Wave Equation

E.V. Krishnan, M. Al Ghabshi, M. Alquran

<http://arxiv.org/abs/1911.10593>

Standard vortex solutions of the extended Painlevé P.D.E

Panayotis Smyrnelis

<http://arxiv.org/abs/1911.10944>

Green's function of the screened Poisson's equation on the sphere

Ramy Tanios, Samah El Mohtar, Omar Knio, Issam Lakkis

<http://arxiv.org/abs/1911.12708>

Generalised Kähler Structure on  $\mathbb{C}P^2$  and Elliptic Functions

Francesco Bonechi, Jian Qiu, Marco Tarlini

<http://arxiv.org/abs/1911.12787>

Flags of sheaves, quivers and symmetric polynomials

Giulio Bonelli, Nadir Fasola, Alessandro Tanzini

<http://arxiv.org/abs/1912.00500>

Riemann–Hilbert Characterisation of Rational Functions with a General Distribution of Poles on the Extended Real Line Orthogonal with Respect to Varying Exponential Weights: Multi-Point Padé Approximants and Asymptotics

A. Vartanian, X. Zhou

<http://arxiv.org/abs/1912.00538>

Symmetries of the space of solutions to special double confluent Heun equation of negative integer order and its applications

Sergey I. Tertychniy

<http://arxiv.org/abs/1912.00663>

Some supercongruences arising from symbolic summation

Ji-Cai Liu

<http://arxiv.org/abs/1912.01035>

Periodic Pólya Urns, the Density Method, and Asymptotics of Young Tableaux

Cyril Banderier, Philippe Marchal, Michael Wallner

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The 2-Adic Analysis of Stirling Numbers of the Second Kind via Higher Order Bernoulli Numbers and polynomials

Arnold Adelberg

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Riordan arrays, the  $A$ -matrix, and Somos 4 sequences

Paul Barry

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A new method of constructing Askey–Wilson type integral

Chuanan Wei

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Zeta functions of  $Z_p$ -towers of curves  
Daqing Wan

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Rank-Metric Codes, Generalized Binomial Moments and their Zeta Functions  
Eimear Byrne, Giuseppe Cotardo, Alberto Ravagnani

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On the Gaussian functions of two discrete variables  
Nicolae Cotfas

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A combinatorial construction for two formulas in Slater's List  
Kağan Kurşungöz

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Manuel Bello-Hernández, Héctor Pijeira-Cabrera, Daniel Rivero-Castillo

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Ravshan Ashurov

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Can polylogarithms at algebraic points be linearly independent?  
Sinnou David, Noriko Hirata-Kohno, Makoto Kawashima

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Heterogeneous hypergeometric functions with two matrix arguments and the exact distribution of the largest eigenvalue of a singular beta-Wishart matrix  
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On The Complex Zeros of The Riemann Zeta Function  
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Explicit fundamental solution for the operator  $L + \alpha|T|$  on the Gelfand pair  $(\mathbb{H}_n, U(n))$   
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Ushangi Goginava, Giorgi Oniani

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The first moment of Maass form symmetric square  $L$ -functions  
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Multiple zeta values and multiple Apéry-like sums  
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Alexander Sakhnovich

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Miranda C. N. Cheng, Francesca Ferrari, Gabriele Sgroi

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Jan Hendrik Bruinier, Markus Schwagenscheidt

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A weighted central limit theorem for  $\log |\zeta(\frac{1}{2} + it)|$   
Alessandro Fazzari

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Schwarz’s map for Appell’s second hypergeometric system with quarter integer parameters  
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V́ctor Almeida, Jorge J. Betancor, Estefanía Dalmaso, Lourdes Rodŕguez–Mesa

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Completions and algebraic formulas for the coefficients of Ramanujan’s mock theta functions  
David Klein, Jennifer Kupka

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Confluent hypergeometric expansions of the confluent Heun function governed by two–term recurrence relations  
T. A. Ishkhanyan, V. P. Krainov, A. M. Ishkhanyan

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On the conditions for a special entire function relative to the partial theta–function and the Euler function to belong to the Laguerre–Pólya class

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Zero Distribution of  $v$ -adic Multiple Zeta Values over  $\mathbb{F}_q(t)$

Qibin Shen

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Exponential integral representations of theta functions

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Geometric properties of Clausen’s Hypergeometric Function  ${}_3F_2(a, b, c; d, e; z)$

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Gamma function solutions to the star–triangle equation

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The analytic theory of indefinite zeta functions

Gene S. Kopp

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A note on Mellin transform, Eisenstein Series and distribution  $d\varepsilon_{it}$  on  $PSL(2, \mathbb{Z}) \backslash PSL(2, \mathbb{C})$

Otto Romero

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Recurrence Relations of the Multi–Indexed Orthogonal Polynomials VI : Meixner–Pollaczek and continuous Hahn types

Satoru Odake

<http://arxiv.org/abs/1912.12445>

Congruence properties of coefficients of the eighth order mock theta function  $V_0(q)$

B. Hemanthkumar

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Dualities for rational multi-particle Painlevé systems: Spectral versus Ruijsenaars  
Ilia Gaiur, Vladimir Rubtsov

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Interlacing Properties of Coefficient Polynomials in Differential Operator Representations of Real-Root Preserving Linear Transformations  
David A. Cardon, Evan L. Sorensen, Jason C. White

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Frobenius structures on hypergeometric equations  
Kiran S. Kedlaya

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Multiplicity-free  $U_q(\mathfrak{sl}_N)$  6- $j$  symbols: relations, asymptotics, symmetries  
Victor Alekseev, Andrey Morozov, Alexey Sleptsov

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On a Family of Hypergeometric Polynomials  
Kikunga Kasenda Ivan

Topic #9 ——— OP – SF Net 27.1 ——— January 15, 2020

From: OP–SF Net Editors

Subject: Submitting contributions to OP–SF NET and SIAM–OPSF (OP–SF Talk)

To contribute a news item to OP–SF NET, send e–mail to one of the OP–SF Editors [howard.cohl@nist.gov](mailto:howard.cohl@nist.gov), or [spost@hawaii.edu](mailto:spost@hawaii.edu).

Contributions to OP–SF NET 27.2 should be sent by March 1, 2020.

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SIAM–OPSF (OP–SF Talk) is a listserv of the SIAM Activity Group on Special Functions and Orthogonal Polynomials, which facilitates communication among members, and friends of the Activity Group. See the previous Topic. To post an item to the listserv, send e–mail to [siam-opsf@siam.org](mailto:siam-opsf@siam.org).

WWW home page of this Activity Group:

<http://math.nist.gov/opsf>

Information on joining SIAM and this activity group: [service@siam.org](mailto:service@siam.org)

The elected Officers of the Activity Group (2020–2022) are:

Peter Alan Clarkson, Chair

Luc Vinet, Vice Chair

Andrei Martínez–Finkelshtein, Program Director

Teresa E. Pérez, Secretary

The appointed officers are:

Howard Cohl, OP–SF NET co–editor

Sarah Post, OP–SF NET co–editor

Diego Dominici, OP–SF Talk moderator

Bonita Saunders, Webmaster and OP–SF Talk moderator

Topic #10 ——— OP – SF Net 27.1 ——— January 15, 2020

From: OP–SF Net Editors

Subject: Thought of the Month by **Francesco Giacomo Tricomi**

*“Mi auguro che, se anche non fossi sempre riuscito a rendere facili le cose difficili, almeno non mi si trovi mai colpevole di rendere artificialmente difficili le cose facili!”*

“Even if I may not have always succeeded in simplifying difficult things, I hope that no one will ever find me guilty of artificially complicating easy things!”

Francesco Giacomo Tricomi, Torino, 1 July 1959. Quoted in the preface of *Esercizi e Complementi di Analisi Matematica Parte Seconda, Terza Edizione*, CEDAM, Padova (1960).

*Contributed by Francesco Mainardi.*