# O P-S F N E T – Volume 32, Number 4 – July 15, 2025

# The Electronic News Net of the SIAM Activity Group on Orthogonal Polynomials and Special Functions

# http://math.nist.gov/opsf

OP-SF Net is distributed to OPSF Activity Group members and non-members alike through the OP-SF Talk listserv.

If you are interested in subscribing to the Newsletter and/or OP-SF Talk, or if you would like to submit a topic to the Newsletter or a contribution to OP-SF Talk, please send an email to the OP-SF Net Editors.

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# Calendar of Events:

# July 28-August 1 , 2025

The Third Joint SIAM/CAIMS Annual Meetings (AN25) Montréal, Québec, Canada https://www.siam.org/conferences-events/siam-conferences/an25/

# August 6-7, 2025

2<sup>nd</sup> International Conference on Mathematical Analysis and Applications (MAA 2025) with Virtual / Online Presentations National Institute of Technology Jamshedpur, Jamshedpur, India https://maanitjsr.github.io/

# August 19-22, 2025

Extremal Polynomials and Dynamical Systems Carlsberg Academy, Copenhagen, Denmark https://www.math.ku.dk/english/calendar/events/epds/

# June 22-26, 2026

OPERA 2026 - Orthogonal Polynomials, Exponential analysis, Rational Approximation, with applications University of Stirling, Scotland, UK https://www.opera2026.uk/

# July 8-18, 2026

Foundations of Computational Mathematics (FoCM 2026), University of Vienna, Vienna, Austria https://focm2026.univie.ac.at/

Workshop related to SIAG/OPSF:

July 9–11: **Special Functions and Orthogonal Polynomials** Organizers: Daan Huybrechs, Erik Koelink and Teresa Pérez

# August 17-21, 2026

18<sup>th</sup> International Symposium on Orthogonal Polynomials, Special Functions and Applications Muromachi Campus, Doshisha University, Kyoto, Japan

Topic #1 \_\_\_\_\_ OP - SF Net 32.4 \_\_\_\_\_ July 15, 2025

From: Manuel Domínguez de la Iglesia (manuel.dominguezi@uah.es) Subject: Announcement: OPSF-S11 Summer School in Alcalá, Spain

The OPSF-S11 Summer School (see link for other OPSFA summer schools in this series) will take place at the Universidad de Alcalá, Spain (near Madrid) from June 15 to June 19, 2026. This edition offers a comprehensive program on orthogonal polynomials and special functions, combining classical topics with modern developments and applications. The school includes lectures, tutorials, and research sessions, and is intended for Master's, PhD, and postdoctoral researchers with a solid background in mathematics, physics, or related disciplines.

The summer school will host five different sets of courses/lectures taught by: (1) Marco Bertola (Concordia University, Montreal, Canada), (2) Annie Cuyt (University of Stirling, Scotland, UK, and Universiteit Antwerpen, Belgium), (3) Arno Kuijlaars (Katholieke Universiteit Leuven, Belgium), (4) Lidia Fernández (Universidad de Granada, Spain) and (5) Luis Velázquez (Universidad de Zaragoza, Spain).

The course descriptions are as follows:

• **Course 1**: Asymptotic methods for Orthogonal Polynomials and more general Padé approximants.

Lecturer: Marco Bertola (Concordia University, Montreal, Canada).

**Course overview**: The goal of the course is to familiarize the students with the description of the asymptotic of orthogonal polynomials that employs the so-called "Riemann Hilbert Method". The method was introduced in the late nineties and saw the main application in the first proof of "universality" results in Random Matrix Theory. In the literature, the specific technique is variably referred to as "nonlinear steepest descent method" or "Deift-Zhou method". A particular appeal of the method is that it applies to a wider range of problems which include asymptotic studies for nonlinear integrable wave equations like the Korteweg-deVries equation or the nonlinear Schrödinger equation, used in oceanography and nonlinear optics, respectively. The goal of the course is, however, focused on the case of orthogonal polynomials, or rather denominators of Padé approximations (namely the so-called "non-Hermitean" orthogonality). Of special interest are the cases of "semiclassical" orthogonal polynomials introduced by Shohat, Maroni, Marcellán, Rocha, and their asymptotics. The method combines input from potential theory as well as analysis, and is able to provide "strong-asymptotic results" for orthogonal polynomials, in principle allowing for a complete asymptotic expansion for large degree valid point-wise in the plane. Time permitting, we will touch upon applications of the method beyond orthogonal polynomials and to the newly introduced notion of Padé approximations on higher genus Riemann surfaces.

A rough breakdown of the course is as follows:

- Padé approximations and Orthogonal Polynomials;
- Generalities about Riemann-Hilbert problems (RHPs); formulation of Padé approximation problems in terms of RHPs.
- Large degree asymptotic analysis:
  - 1. Elements of potential theory and the "g-function" mechanism;
  - 2. Reduction of the RHP to a "small-norm" problem;
  - 3. Construction of "parametrices"; approximate solutions;
  - Some aspects of geometry of Riemann surfaces needed for the construction of parametrices;
  - 5. Padé approximants on Riemann surfaces (time permitting);
  - 6. Other applications of the method and interesting problems (time permitting)

The method is seeing continued developments and new applications to this day, with an ever growing literature; the first instances where the method was developed are:

- P. Deift and X. Zhou, A steepest descent method for oscillatory Riemann-Hilbert problems. Bull. Amer. Math. Soc. (N.S.), 26(1):119-123, 1992.
- P. Deift, "Orthogonal Polynomials and Random Matrices: a Riemann-Hilbert approach"; Courant Institute Lectures, ('98);
- P. Deift, T. Kriecherbauer, K. T.-R. McLaughlin, S. Venakides, and X. Zhou, Uniform asymptotics for polynomials orthogonal with respect to varying exponential weights and applications to universality questions in random matrix theory. Comm. Pure Appl. Math., 52(11):1335-1425, 1999.
- **Course 2**: Connections between Orthogonal Polynomials, Sparse Interpolation, Exponential Analysis, Padé Approximation and Gaussian Integration.

Lecturer: Annie Cuyt (University of Stirling, Scotland, UK, and Universiteit Antwerpen, Belgium).

**Course overview**: This course explores the deep connections between various mathematical tools and techniques: orthogonal polynomials, Padé approximation, Gaussian quadrature, exponential analysis, and sparse interpolation. Despite appearing in different contexts, all of these methods are linked through a common structure—Hankel matrices.

The course explains how orthogonal polynomials can be formally defined using a linear functional, and how these polynomials naturally relate to Padé approximants of a power series with the linear functional's scalar values as coefficients. When, in addition, these values are moment integrals with respect to a weight function, the Padé approximants can be interpreted as Gaussian quadrature rules, where the nodes and weights have a direct connection to the polynomials orthogonal with respect to the given linear functional.

Further, the problem of reconstructing the Gaussian nodes and weights from the given moments, is shown to be equivalent to exponential analysis or the Prony problem—a classic inverse problem. In the context of computer algebra, this is referred to as sparse interpolation. The discussion also highlights how the nodes can be obtained from the generalized eigenvalues of a Hankel structured matrix pencil, and how the weights are computed by solving a linear system with a Vandermonde structure. Finally, it is noted that these relationships and methods can be extended to the multivariate setting, provided the appropriate generalizations are applied.

• **Course 3**: From Jacobi polynomials to random tilings.

Lecturer: Arno Kuijlaars (Katholieke Universiteit Leuven, Belgium).

**Course overview**: In the first part of the course, we discuss the asymptotic behavior of Jacobi polynomials with varying parameters. This is used as a toy model to illustrate the powerful Riemann-Hilbert analysis The second part of the course will cover a recent application of Jacobi polynomials with varying non-standard parameters to random tiling problems.

# Provisional outline:

- 1. Jacobi polynomials with classical parameters
- 2. Riemann-Hilbert problem
- 3. Jacobi polynomials with non-standard parameters
- 4. Lozenge tilings of hexagon
- 5. Eynard-Mehta formula
- 6. Asymptotics of Jacobi polynomials.
- **Course 4**: Multivariate orthogonal polynomials and applications.

Lecturer: Lidia Fernández (Universidad de Granada, Spain).

**Course overview**: Multivariate orthogonal polynomials are not just a simple generalization of the polynomials in a single variable, but they are actually very complex mathematical objects with singular properties. The first part of the course will cover the general properties of orthogonal polynomials in several variables including, for example, recurrence relations, Jacobi matrices or Christoffel–Darboux formulae. Some interesting examples will be analyzed, pointing out the different bases and their particular properties. The final part of the course will focus the applications of these polynomials, such as the relation with optics and aberrations.

• **Course 5**: Diagrammatic and harmonic analysis methods for orthogonal polynomials.

Lecturer: Luis Velázquez (Universidad de Zaragoza, Spain).

**Course overview**: Orthogonal polynomials have links with the study of random walks -more generally, Markov chains- which have been traditionally used to get probabilistic information about such random systems via standard methods from the theory of orthogonal polynomials. These lectures will show some payoffs of this connection which take the form of diagrammatic techniques in orthogonal polynomial theory. Among other things, this diagrammatic approach sheds light on another classical connection which links orthogonal polynomials and harmonic analysis, providing new results in both areas. The course will show some of these results, as well as some open problems posed by this new look at orthogonal polynomial theory.

# Organizing Committee:

- Amílcar Branquinho, Universidade de Coimbra, Portugal
- Manuel Domínguez de la Iglesia, Universidad de Alcalá, Spain
- Ana Foulquié Moreno, Universidade de Aveiro, Portugal
- Edmundo Huertas Cejudo, Universidad de Alcalá, Spain
- Alberto Lastra Sedano, Universidad de Alcalá, Spain
- Manuel Mañas Baena, Universidad Complutense de Madrid, Spain

For more information please contact: opsf.s11@uah.es. A dedicated website will be available soon.

Topic #2 \_\_\_\_\_ OP - SF Net 32.4 \_\_\_\_\_ July 15, 2025

From: Annie Cuyt (annie.cuyt@uantwerpen.be) Subject: Announcement: OPERA 2026 Conference in Stirling, Scotland

OPERA: Orthogonal Polynomials, Exponential analysis, Rational Approximation, with Applications

Location: University of Stirling, Scotland, UK

Dates: June 22-26, 2026

We are delighted to invite you to OPERA 2026, a conference aiming to bring together researchers working on topics and applications related to orthogonal polynomials, exponential analysis, and rational approximation. These include, but are not limited to, signal processing applications, computer algebra methods, nonlinear approximation theory, structured matrices and subdivision schemes.

# Plenary Speakers:

- Bernard Beckermann, University of Lille, France
- Stefano De Marchi, University of Padua, Italy
- Stefan Güttel, University of Manchester, UK
- Miguel Piñar, University of Granada, Spain
- Gerlind Plonka-Hoch, University of Göttingen, Germany

# Scientific Committee:

- Pier Luigi Dragotti, Imperial College London
- Wen-shin Lee, University of Stirling
- David Li, University of Strathclyde
- Ana Loureiro, University of Kent

Stirling, known as the "heart of Scotland" and "Gateway to the Highlands", is easily accessible from either Edinburgh or Glasgow.

Stay tuned for updates and registration information at opera2026.uk.

Topic #3 \_\_\_\_\_ OP - SF Net 32.4 \_\_\_\_\_ July 15, 2025

From: Mark MacLean (macleanm@seattleu.edu) Subject: Report: TerwilligerFest by MacLean

Report on the Combinatorics around the *q*-Onsager Algebra Conference.

In the last week of June, mathematicians from around the globe convened for the Combinatorics around the *q*-Onsager Algebra Conference in Kranjska Gora, Slovenia. This conference, also called Terwilliger-FEST, was a celebration of the many significant mathematical contributions of Paul Terwilliger on the occasion of his 70<sup>th</sup> birthday.

The conference featured 46 different speakers, whose talks spanned the wide range of topics Paul's work has touched on over the years, including distance-regular graphs, association schemes, Leonard pairs,



Figure 1: Organizing and scientific committee members: (Top row) Giusy Monzillo, Mark MacLean, Paul Terwilliger, Stefko Miklavic; (bottom row) Rene Rodriguez Aldama, Blas Fernandez, Safet Penjic.

orthogonal polynomials from the Askey scheme, and the q-Onsager algebra. The full list of the speakers and abstracts can be found at the conference website, https://conferences.famnit.upr.si/event/32/overview.

Beyond the academic discussions, attendees enjoyed the natural beauty of Slovenia, with the conference lecture hall offering stunning floor-to-ceiling views of the Julian Alps. A highlight of the conference was Paul's talk at the banquet, entitled "Fifteen breakthroughs in algebraic combinatorics." He outlined fifteen pivotal mathematical advancements by his peers that profoundly influenced his own research over the course of his career. The evening concluded with many participants sharing touching personal stories about Paul's immense impact on their lives and work.

A special issue of the *Journal of Algebraic Combinatorics* will be devoted to the proceedings of the conference. The organizers would like to extend our gratitude to our sponsors for their financial support: the University of Primorska, the Slovenian Discrete and Applied Mathematics Society, and the Institute of Mathematics, Physics and Mechanics.

Topic #4 \_\_\_\_\_ OP - SF Net 32.4 \_\_\_\_\_ July 15, 2025

From: Sergei Suslov (sergei@asu.edu) Subject: Essay: " $e^{\pi} \approx \pi^{e}$ , a brief remark" by **Turbiner** and **Znojil** 

 $e^{\pi} \approx \pi^{e}$ , a brief remark

by: Alexander V. Turbiner and Miloslav Znojil



Figure 2: Group photo of TerwilligerFEST.

This work is dedicated to the memory of the remarkable Czech mathematical physicist Miloslav Havlícek-the exemplary scientist and citizen.

#### Introduction

The habit of almost all ancient civilizations of denoting numbers by letters survived for a fairly long time. The best example is the ancient Greek numeration system in which the integer value of 1 was denoted by the first letter of the alphabet (with a prime),  $\alpha' = 1$  and, similarly,  $\beta' = 2$ , etc.

One of the very last remnants of such a habit (but using *different* alphabets!) are the numbers  $\pi \approx 3.141592654...$  and  $e \approx 2.718281828...$  Several persuasive reasons can be found for this. First of all, perhaps, a direct (or indirect) reference has to be made to the famous "Euler's formula" (see also link) [1],

$$e^{i\pi} + 1 = 0, (1)$$

where i is the imaginary unit. While the numerical values of 0, 1 and i are known exactly, both of the transcendentals e and  $\pi$  are only available via an *ad hoc* approximation. Their exact explicit specification is impossible because it would require the knowledge of infinitely many decimal digits. This makes the formula mind-boggling. Second of all, if (-1) in (1) is replaced by  $i^2$ , we arrive at

$$e^{i\pi} = i^2 . \tag{2}$$

This can be interpreted as a relation between  $e, \pi$  and i. A natural question to ask is whether there exists a relation between e and  $\pi$ , if i is dropped.

#### Peculiarity of e and $\pi$ , main formula

It is truly striking that the numerical values of  $\pi \approx 3.141592654$  and  $e \approx 2.718281828$  do not in fact lie too far away from each other. Quantitatively, their relative difference

$$2\frac{\pi - e}{\pi + e} = 0.14447778\dots,$$
(3)

is comparatively small, about 14%.

Naturally, one must treat this as just an accident and, moreover, about not too striking a coincidence. Still, in our eyes, the situation has thoroughly changed when we more recently noticed that another and truly inspiring parallel surprise emerges when one turns one's attention to the numerical evaluation of the expressions  $e^{d\pi}$ , for d equal to the smallest Heegner number 1, with

$$e^{\pi} = 23.1406926328\dots$$
 (4)

Feeling, not quite expectedly, encouraged by the existence of a part of the decimal expansion of such an exponent in (cf. OEIS sequence A039661) as well as by the existence of its dedicated name (viz., "Gelfond's constant"), we finally found it interesting that expression (4) does not in fact lie too far from its "relative"

$$\pi^e = 22.4591577\dots$$
 (5)

(see also OEIS sequence A059850). Intuitively, we initially expected that the absolute value of the difference  $e^{\pi} - \pi^{e}$  will be very large. Nevertheless, to our great surprise we revealed that the relative difference drops down below 3%,

$$2\frac{e^{\pi} - \pi^{e}}{e^{\pi} + \pi^{e}} = 0.02989198\dots,$$
(6)

cf. (3). We could not resist the impression that the fivefold decrease of the relative difference should be interpreted as simply miraculous. Eventually, we arrive at the approximate equality

$$e^{\pi} \approx \pi^{e}$$
, (7)

which holds within 3%. It can be interpreted as a relation between e and  $\pi$ .

Surprisingly, the relation (7) is absent in the Ramanujan notebooks  $[2]^1$ . Note that (4) is always larger than  $(5)^2$ . By taking the natural logarithm of the left-hand side and right-hand side of (7), we arrive at an even more accurate approximate equality,

$$\pi \approx e \log_e \pi = 3.111698447...,$$
 (8)

where the relative difference

$$2\frac{\pi - e \log_e \pi}{\pi + e \log_e \pi} = 0.00956111\dots,$$
(9)

cf. (3), (6), is less than 1%. Note that from (8) an approximate representation for the number *e* follows,

$$e \approx \frac{\pi}{\log_e \pi} = 2.744396466\dots,$$
 (10)

which holds with the same relative difference (9). Note that by taking the logarithm of the left-hand side and right-hand side of (10) we arrive at the approximate representation for the unity,

$$1 \approx \log_e \pi - \log_e \log_e \pi = 1.009561184....$$
 (11)

<sup>&</sup>lt;sup>1</sup>We thank Prof. B. C. Berndt for this information.

<sup>&</sup>lt;sup>2</sup>If we take two functions  $f_1(x) = e^x$  and  $f_2(x) = x^e$ : for x = e they coincide, however, with growth x > e, it is always  $f_1 > f_2$  and their relative difference will slowly increase with x, in particular, for  $x = \pi$  the relative difference reaches (6).

# Conclusions

The approximate but highly accurate relations (7), (8), (10) and (11) of the Ramanujan style between e and  $\pi$  are presented. Similarly to many relations found by Ramanujan it is unclear how to make our relations more accurate. Such relations could be of natural use while making estimates in science and engineering.

#### Acknowledgments

AVT thanks Czech Technical University for kind hospitality where this work was finished. AVT wants to express gratitude to W. Bietenholz and J. C. Lopez Vieyra (ICN, UNAM, Mexico) for the interest to the subject and interesting email communications. AVT is partially supported by DGAPA grant IN104125 (Mexico).

#### Bibliography

[1] Complex Analysis. Lars V. Ahlfors, McGraw-Hill Book Co., New York, 1978.

[2] Ramanujan's Notebooks. Parts I-V, Bruce C. Berndt, Springer-Verlag, New York, 1985.

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Topic #5 \_\_\_\_\_ OP - SF Net 32.4 \_\_\_\_\_ July 15, 2025

From: OP-SF Net Editors Subject: Preprints in arXiv.org

The following preprints related to the fields of orthogonal polynomials and special functions were posted or cross-listed to one of the subcategories of arXiv.org during May and June 2025. This list has been separated into two categories.

# **OP-SF Net Subscriber E-Prints**

#### http://arxiv.org/abs/2505.00381

Proximal gradient-type method with generalized distance and convergence analysis without global descent lemma Shotaro Yaqishita, Masaru Ito

#### http://arxiv.org/abs/2505.01344

Multiplicative congruences for Andrews's even parts below odd parts function and related infinite products Frank Garvan, Connor Morrow

#### http://arxiv.org/abs/2505.02029

Efficient computation of soliton gas primitive potentials Cade Ballew, Deniz Bilman, Thomas Trogdon

 $\mathbb{Z}_2^3-$ grading of the Lie algebra  $G_2$  and related color algebras N. I. Stoilova, J. Van der Jeugt

http://arxiv.org/abs/2505.05005

A note on the irrationality of  $_2(5)$  Li Lai, Johannes Sprang, Wadim Zudilin

#### http://arxiv.org/abs/2505.05342

Rigorous Methods for Bohr-Sommerfeld Quantization Rules Joanne Dong, Peter D. Miller, Giorgio Young

#### http://arxiv.org/abs/2505.05574

Summation formulas for Hurwitz class numbers and other mock modular coefficients Olivia Beckwith, Nikolaos Diamantis, Rajat Gupta, Larry Rolen, Kalani Thalagoda

#### http://arxiv.org/abs/2505.06788

Two quantitative versions of the Nonlinear Carleson Conjecture Sergey A. Denisov

#### http://arxiv.org/abs/2505.06830

New systems of log-canonical coordinates on SL(2) character varieties of compact Riemann surfaces Marco Bertola, Dmitry Korotkin, Jordi Pillet

#### http://arxiv.org/abs/2505.08099

Signed Partitions and Rogers-Ramanujan type Identities Abdulaziz M. Alanazi, Augustine O. Munagi, Andrew V. Sills

#### http://arxiv.org/abs/2505.09727

Accelerating Fast Ewald Summation with Prolates for Molecular Dynamics Simulations Jiuyang Liang, Libin Lu, Alex Barnett, Leslie Greengard, Shidong Jiang

#### http://arxiv.org/abs/2505.11956

Zeros of linear combinations of orthogonal polynomials Antonio J. Durán

#### http://arxiv.org/abs/2505.12633

Asymptotics for a class of planar orthogonal polynomials and truncated unitary matrices Alfredo Deaño, Kenneth T-R McLaughlin, Leslie Molag, Nick Simm

#### http://arxiv.org/abs/2505.14441

Orientation Reversal and the Chern-Simons Natural Boundary Griffen Adams, Ovidiu Costin, Gerald V. Dunne, Sergei Gukov, Oğuz Öner

#### http://arxiv.org/abs/2505.15330

Zeros of linear combinations of Hermite polynomials Antonio J. Durán

#### http://arxiv.org/abs/2505.21917

Structured Divide-and-Conquer for the Definite Generalized Eigenvalue Problem James Demmel, Ioana Dumitriu, Ryan Schneider

Rigidity of surfaces with nonpositive Euler characteristic by the second eigenvalue of the Jacobi operator Márcio Batista, Marcos P. Cavalcante, Abraão Mendes, Ivaldo Nunes

#### http://arxiv.org/abs/2505.22588

Overpartitions and Kaur, Rana, and Eyyunni's mex sequences Brian Hopkins, James A. Sellers

#### http://arxiv.org/abs/2505.22896

Exploring Integration by Differentiation R. D. George, C. Vignat

#### http://arxiv.org/abs/2505.24530

A Combinatorial Study of the Fixed Point Index Jesús A. Álvarez López, Alejandro O. Majadas-Moure, David Mosquera-Lois

#### http://arxiv.org/abs/2506.00810

On the average scale-invariant Cassinian metric Manas Mohapatra, Antti Rasila, Matti Vuorinen

#### http://arxiv.org/abs/2506.01865

Some series connecting Fibonacci numbers to Zhi-Wei Sun, Yajun Zhou

http://arxiv.org/abs/2506.01886 Two 2/5-level mock theta conjecture-like identities Stepan Konenkov, Eric T. Mortenson

#### http://arxiv.org/abs/2506.02190

2-Homogeneous bipartite distance-regular graphs and the quantum group  $U_q'(\mathfrak{so}_6)$ Paul Terwilliger

#### http://arxiv.org/abs/2506.02424

An adaptive delaminating Levin method in two dimensions Shukui Chen, Kirill Serkh, James Bremer, Murdock Aubry

#### http://arxiv.org/abs/2506.03003

Newtonian potentials of Legendre polynomials on rectangles have displacement structure Sheehan Olver

#### http://arxiv.org/abs/2506.03432

Differential equations for a class of semiclassical orthogonal polynomials on the unit circle Cleonice F. Bracciali, Karina S. Rampazzi, Luana L. Silva Ribeiro

#### http://arxiv.org/abs/2506.04918

Orthogonality of polar Legendre polynomials and approximation Abdelhamid Rehouma

#### http://arxiv.org/abs/2506.05013

Generalized product formulas for Whittaker's functions and a novel class of index transforms Semyon Yakubovich

Infinitesimal freeness for orthogonally invariant random matrices Guillaume Cébron, James A. Mingo

#### http://arxiv.org/abs/2506.05492

Zeros of orthogonal little q-Jacobi polynomials: interlacing and monotonicity Andrei Martinez-Finkelshtein, Rafael Morales, Daniel Perales

#### http://arxiv.org/abs/2506.05622

Deformations of OP ensembles in a bulk critical scaling Caio E. Candido, Victor Alves, Thomas Chouteau, Charles F. Santos, Guilherme L. F. Silva

#### http://arxiv.org/abs/2506.05861

Cubic graphs with no eigenvalues in the interval (-2,0) Krystal Guo, Gordon F. Royle

#### http://arxiv.org/abs/2506.05961

Generalization of Ramanujan's formula for the sum of half-integer powers of consecutive integers via formal Bernoulli series Max A. Alekseyev, Rafael Gonzalez, Keryn Loor, Aviad Susman, Cesar Valverde

#### http://arxiv.org/abs/2506.06101

Ramanujan's partition generating functions modulo  $\ell$  Kathrin Bringmann, William Craig, Ken Ono

#### http://arxiv.org/abs/2506.06461

Constructing strong starters of orders 3p: triplication with SAT solver Oleg Ogandzhanyants, Sergey Sadov, Margo Kondratieva

#### http://arxiv.org/abs/2506.06550

A New Two-Sample Test for Covariance Matrices in High Dimensions: U-Statistics Meet Leading Eigenvalues

Thomas Lam, Nina Dörnemann, Holger Dette

#### http://arxiv.org/abs/2506.09015

Flagged LLT polynomials, nonsymmetric plethysm, and nonsymmetric Macdonald polynomials Jonah Blasiak, Mark Haiman, Jennifer Morse, Anna Pun, George H. Seelinger

#### http://arxiv.org/abs/2506.10556

Lambert's problem in orbital dynamics: a self-contained introduction Lenox Helene Baloglou, Parneet Gill, Tonatiuh Sánchez-Vizuet

#### http://arxiv.org/abs/2506.10959

Understanding In-Context Learning on Structured Manifolds: Bridging Attention to Kernel Methods Zhaiming Shen, Alexander Hsu, Rongjie Lai, Wenjing Liao

#### http://arxiv.org/abs/2506.14259

Dense Phenomena for Ergodic Schrödinger Operators: I. Spectrum, Integrated Density of States, and Lyapunov Exponent Artur Avila, David Damanik

Partition function of 2D Coulomb gases with radially symmetric potentials and a hard wall Matthias Allard, Peter J. Forrester, Sampad Lahiry, Bojian Shen

#### http://arxiv.org/abs/2506.14996

Positive m-divisible non-crossing partitions and their Kreweras maps Christian Krattenthaler, Christian Stump

#### http://arxiv.org/abs/2506.17069

Algebra of double cosets of a symmetric group by a smaller symmetric group Yury A. Neretin

#### http://arxiv.org/abs/2506.17178

Hecke polynomials for the mock modular form arising from the Delta-function Kevin Gomez, Ken Ono

#### http://arxiv.org/abs/2506.17684

Pattern formation Statistics on Fermat Quotients Cristian Cobeli, Alexandru Zaharescu, Zhuo Zhang

#### http://arxiv.org/abs/2506.17862

Proofs Of Three Geode Conjectures Tewodros Amdeberhan, Doron Zeilberger

http://arxiv.org/abs/2506.18712 Lambert series and double Lambert series Tewodros Amdeberhan, George E. Andrews, Cristina Ballantine

#### http://arxiv.org/abs/2506.18834

On the random-time and finite-time ruin probability for widely dependent claim sizes and inter-arrival times

Yang Chen, Zhaolei Cui, Yuebao Wang

# http://arxiv.org/abs/2506.20289

(Strange) gamma evaluations Wadim Zudilin

http://arxiv.org/abs/2506.20934

Simplified Airy function Asymptotic expansions for Reverse Generalised Bessel Polynomials T. M. Dunster

http://arxiv.org/abs/2506.21306

On Uniform Weighted Deep Polynomial approximation Kingsley Yeon, Steven B. Damelin

http://arxiv.org/abs/2506.22916

Best approximation by polynomials on the conic domains Yan Ge, Yuan Xu

#### http://arxiv.org/abs/2506.23082

Hall-Littlewood expansions of chromatic quasisymmetric polynomials using linked rook placements Jang Soo Kim, Seung Jin Lee, Meesue Yoo

# Other Relevant OP-SF E-Prints

#### http://arxiv.org/abs/2505.01732

Tesler identities for wreath Macdonald polynomials Marino Romero, Joshua Jeishing Wen

#### http://arxiv.org/abs/2505.02202

Multiple polylogarithms and the Steinberg module Steven Charlton, Danylo Radchenko, Daniil Rudenko

#### http://arxiv.org/abs/2505.03224

Analytic continuation of Kochubei multiple polylogarithms and its applications Yen-Tsung Chen

#### http://arxiv.org/abs/2505.03967 Chebyshev polynomials on equipotential curves Erwin Miña-Díaz, Olof Rubin

#### http://arxiv.org/abs/2505.04039

Dwork congruences via *q*-deformation Pavan Kartik, Andrey Smirnov

#### http://arxiv.org/abs/2505.04395

A refined *q*-analogue of some congruences of Van Hamme Chen Wang, Yu-Chan Tian, Kai Huang

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Expectations of some ratio-type estimators under the gamma distribution Jia-Han Shih

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On a specific family of orthogonal polynomials of Bernstein-Szegő type Martin Nicholson

#### http://arxiv.org/abs/2505.05858

Symmetry of hypergeometric functions over finite fields and geometric interpretation Akio Nakagawa

#### http://arxiv.org/abs/2505.06857 Degenerations of *q*-Heun equation Chihiro Sato, Kouichi Takemura

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The  $\mathbb{Z}$ -module of multiple zeta values is generated by ones for indices without ones Minoru Hirose, Takumi Maesaka, Shin-ichiro Seki, Taiki Watanabe

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Brownian behaviour of the Riemann zeta function around the critical line Louis Vassaux

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The polytope of all *q*-rank functions Gianira N. Alfarano, Sebastian Degen

The "Shape" of *q*-Binomial Coefficients Nate Harman

#### http://arxiv.org/abs/2505.08422

A new bijective proof of the q-Pfaff-Saalschütz identity with applications to quantum groups Álvaro Gutiérrez, Álvaro L. Martínez, Michał Szwej, Mark Wildon

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Calderon's reproducing formula and extremal functions associated with the linear canonical Dunkl wavelet transform Sandeep Kumar Verma, Umamaheswari S

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On the *p*-adic valuations of values of Legendre polynomials Max A. Alekseyev, Tewodros Amdeberhan, Jeffrey Shallit, Ingrid Vukusic

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Some explicit values of a *q*-multiple zeta function at roots of unity Takao Komatsu

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Dynamical zeta functions and resonance chains for infinite-area hyperbolic surfaces with large funnel widths Henry Talbott

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On the critical length conjecture for spherical Bessel functions in CAGD Ognyan Kounchev, Hermann Render

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Ask zeta functions of joins of graphs Tobias Rossmann, Christopher Voll

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The Regular Representation of the twisted queer q-Schur Superalgebra Zhenhua Li

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Inverse nonlinear fast Fourier transform on SU(2) with applications to quantum signal processing Hongkang Ni, Rahul Sarkar, Lexing Ying, Lin Lin

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Two-sided Gaussian estimates for fundamental solutions of second-order parabolic equations in nondivergence form Seick Kim, Sungjin Lee, Georgios Sakellaris

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Tsallis' q-analysis, new scales of interpolating spaces and q-rational functions Daniel Alpay, Paula Cerejeiras, Uwe Kaehler

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Five-Term Relations for wreath Macdonald polynomials and tableau formulas for Pieri coefficients Marino Romero, Joshua Jeishing Wen

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*q*-analogues of Fisher's inequality and oddtown theorem Hiranya Kishore Dey

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Fluctuations of Young diagrams for symplectic groups and semiclassical orthogonal polynomials Anton Nazarov, Anton Selemenchuk

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Characterization of bi-parametric potentials and rate of convergence of truncated hypersingular integrals in the Dunkl setting Sandeep Kumar Verma, Athulya P

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Central limit theorem for the determinantal point process with the confluent hypergeometric kernel Sergei M. Gorbunov

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*d*-orthogonal polynomials, Fuss-Catalan matrices and lattice paths Paul Barry

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Many-faced Painlevé I: irregular conformal blocks, topological recursion, and holomorphic anomaly approaches Nikolai lorgov, Kohei Iwaki, Oleg Lisovyy, Yurii Zhuravlov

#### http://arxiv.org/abs/2505.18974

Sparse domination for singular integral operators and their commutators in Dunkl setting with applications Yanping Chen, Xueting Han

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Bounds for Moments of Dirichlet *L*-functions of fixed modulus on the critical line Peng Gao, Liangyi Zhao

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Discrete harmonic polynomials in multidimensional orthants Emmanuel Humbert, Kilian Raschel

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Symmetries of coefficients of three-term relations for basic hypergeometric series Yuka Yamaguchi

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Rank three representations of Painlevé systems: I. Wild character varieties Miklos Eper, Szilard Szabo

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Fast evaluation of Riemann theta functions in any dimension Noam D. Elkies, Jean Kieffer

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Collapse of the  $\mathfrak{sl}_2$ -triple associated to the (k, a)-generalized Fourier transform Tatsuro Hikawa

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A Closer Look at Chapoton's q-Ehrhart Polynomials Matthias Beck, Thomas Kunze

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Quasi-Homogeneous Integrable Systems: Free Parameters, Kovalevskaya Exponents, and the Painlevé Property Changyu Zhou, Hayato Chiba

On compact sets possessing *q*-convex functions Thomas Pawlaschyk, Nikolay Shcherbina

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On fractional differential equations, dimensional analysis, and the double gamma function J. Vaz, E. Capelas de Oliveira

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Finite version of the *q*-analogue of de Finetti's theorem Adyan Dordzhiev

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Ninth degree analogue of Ramanujan's septic theta function identity Sun Kim, Örs Rebák

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Super Macdonald polynomials and BPS state counting on the blow-up Hiroaki Kanno, Ryo Ohkawa, Jun'ichi Shiraishi

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Jacob's ladders, E. C. Titchmarsh's hypothesis (1934) and new –equivalents of the Fermat–Wiles theorem or connections between Fermat's rationals and the Gram's sequence Jan Moser

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Discrete Painlevé equations from pencils of quadrics in  $\mathbb{P}^3$  with branching generators Jaume Alonso, Yuri B. Suris

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A note on a Pohozaev identity for the fractional Green function Abdelrazek Dieb, Isabella Ianni

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Classification of the real Painlevé I transcendents by zeros and connection problem: an asymptotic study Yan Huang, Yu-Tian Li, Wen-Gao Long

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Gelfand hypergeometric function as a solution to the 2-dimensional Toda-Hirota equation Hironobu Kimura

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Indefinite theta functions arising from affine Lie superalgebras and sums of triangular numbers Toshiki Matsusaka, Miyu Suzuki

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Orthonormal Strichartz estimates for Dunkl-Schrödinger equation of initial data with Sobolev regularity Guoxia Feng, Shyam Swarup Mondal, Manli Song, Huoxiong Wu

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Elliptic asymptotic behaviour of *q*-Painlevé transcendents Joshua Holroyd

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Relative Riemann-Hilbert and Newlander-Nirenberg Theorems for torsion-free analytic sheaves on maximal and homogeneous spaces Thomas Kurbach

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 $q\mathchar`-deformed$  Howe duality for orthosymplectic Lie superalgebras Jeong Bae, Jae–Hoon Kwon

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Fourier transforms and Abel-Jacobi theory Younghan Bae, Sam Molcho, Aaron Pixton

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Green functions and a positive mass theorem for asymptotically hyperbolic 3-manifolds Klaus Kroencke, Francesca Oronzio, Alan Pinoy

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p-adic congruences in iterated derivatives of the Weierstrass elliptic function Kiran Luecke, Eric Peterson

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Modular resurgence, *q*-Pochhammer symbols, and quantum operators from mirror curves Veronica Fantini, Claudia Rella

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Congruence conditions for the mod values of the Fourier coefficients of classical eigenforms Michael A. Daas

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The Fourier transform in variable exponent Lebesgue spaces André Pedroso Kowacs, Wagner Augusto Almeida de Moraes

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Lower bounds for high moments of zeta sums Zikang Dong, Weijia Wang, Hao Zhang

On the exterior power structure of the cohomology groups for the general hypergeometric integral Hironobu Kimura

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Recurrence relations and the Christoffel-Darboux formula for elliptic orthogonal polynomials Harini Desiraju, Sampad Lahiry

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Detecting transitions from steady states to chaos with gamma distribution Haiyan Wang, Ying Wang

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Asymptotic behavior of the fundamental solution of space-time fractional equations with a reaction term Luciano Abadías, Claudio Carrasco, Juan C. Pozo

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Gelfand hypergeometric functions as solutions to the 2-dimensional Toda-Hirota equations II Hironobu Kimura

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Extreme values of derivatives of the Dedekind zeta function of a cyclotomic field Zhonghua Li, Yutong Song, Qiyu Yang, Shengbo Zhao

#### http://arxiv.org/abs/2506.14822

Analysis and conditional optimization of projection estimates for the distribution of random variable using Legendre polynomials Tatyana A. Averina, Konstantin A. Rybakov

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A polynomial projective algorithm for convex feasibility problems with positive-definite constraints Sergei Chubanov

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Dirichlet *L*-functions on the critical line and multiplicative chaos Sami Vihko

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An identity for generating series of deformations of multiple zeta values within an algebraic framework Yoshihiro Takeyama

#### http://arxiv.org/abs/2506.16711

Some q-transformation formulas and Rogers-Ramanujan type identities Chang Xu, Dunkun Yang

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On the error term of the fourth moment of the Riemann zeta-function Neea Palojärvi, Tim Trudgian

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A Generic Construction of q-ary Near-MDS Codes Supporting 2-Designs with Lengths Beyond q+1Hengfeng Liu, Chunming Tang, Zhengchun Zhou, Dongchun Han, Hao Chen

Refined uncertainty relation for *q*-commutator Kenjiro Yanagi

# http://arxiv.org/abs/2506.17416

Explicit conditional bounds for the residue of a Dedekind zeta-function at s=1 Stephan Ramon Garcia, Ethan Simpson Lee

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Algorithms for pointwise and piecewise polynomial approximations to the trigonometric functions Quan Le Phuong

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The second moment of Ramanujan sums Hong Ziwei, Zheng Zhiyong

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Multidimensional vector-valued Laplace transform and applications Marko Kostic

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New zero-free regions for Dedekind zeta-functions at small and large ordinates Sourabhashis Das, Swati Gaba, Ethan Simpson Lee, Aditi Savalia, Peng-Jie Wong

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Krylov and core transformation algorithms for an inverse eigenvalue problem to compute recurrences of multiple orthogonal polynomials Amin Faghih, Michele Rinelli, Marc Van Barel, Raf Vandebril, Robbe Vermeiren

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Values at non-positive integers of partially twisted multiple zeta-functions II Driss Essouabri, Kohji Matsumoto, Simon Rutard

# http://arxiv.org/abs/2506.21092

On the geometry of a 4-dimensional extension of a q-Painlevé I equation with symmetry type  $A_1^{(1)}$ Alexander Stokes, Tomoyuki Takenawa, Adrian Stefan Carstea

# http://arxiv.org/abs/2506.22014

On twisted period functions and Moments of a weighted mean square of Dirichlet L-functions on the critical line Sebastien Darses, Berend Bingeling, Emmanuel Pover

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# http://arxiv.org/abs/2506.22256

Mean squares of quadratic twists of the Fourier coefficients of modular forms Peng Gao, Yuetong Zhao

# http://arxiv.org/abs/2506.23373

The monomial expansions of modified Macdonald polynomials Emma Yu Jin, Xiaowei Lin

Little q-Jacobi polynomials and symmetry breaking operators for  $U_q(sl_2)$ Quentin Labriet, Loïc Poulain d'Andecy

#### http://arxiv.org/abs/2506.24116

On the zero sets of harmonic polynomials Ioann Vasilyev

# Topic #6 \_\_\_\_\_ OP – SF Net 32.4 \_\_\_\_\_ July 15, 2025

From: OP-SF Net Editors Subject: Submitting contributions to OP-SF NET and SIAM-OPSF (OP-SF Talk)

To contribute a news item to OP-SF NET, send e-mail to one of the OP-SF Editors howard.cohl@nist.gov, or spost@hawaii.edu.

Contributions to OP-SF NET 32.5 should be sent by September 1, 2025.

OP-SF NET is the electronic newsletter of the SIAM Activity Group on Special Functions and Orthogonal Polynomials (SIAG/OPSF). We disseminate your contributions on anything of interest to the special functions and orthogonal polynomials community. This includes announcements of conferences, forthcoming books, new software, electronic archives, research questions, and job openings as well as news about new appointments, promotions, research visitors, awards and prizes. OP-SF Net is transmitted periodically through a post to OP-SF Talk which is currently managed and moderated by Howard Cohl (howard.cohl@nist.gov). Anyone wishing to be included in the mailing list (SIAG/OPSF members and non-members alike) should send an email expressing interest to him. Bonita Saunders also posts the Newsletter through SIAM Engage (SIAG/OPSF) which is received by all SIAG/OPSF members.

OP-SF Talk is a listserv associated with SIAG/OPSF which facilitates communication among members, non-members and friends of the Activity Group. To post an item to the listserv, send e-mail to howard.cohl@nist.gov.

WWW home page of this Activity Group: http://math.nist.gov/opsf Information on joining SIAM and this activity group: service@siam.org

The elected Officers of the Activity Group (2025-2027) are: Howard Cohl, Chair Kerstin Jordaan, Program Director Tom Trogdon, Secretary

The appointed officers are: Howard Cohl, OP-SF NET co-editor Sarah Post, OP-SF NET co-editor Bonita Saunders, Webmaster and SIAM Engage (SIAG/OPSF) moderator

Topic #7 \_\_\_\_\_ OP - SF Net 32.4 \_\_\_\_\_ July 15, 2025

From: OP-SF Net Editors Subject: Thought of the Month from **Haiti** 

"Dèyè mòn, gen mon"

English translation: "Beyond mountains, more mountains"

Haitian proverb.