

THE SEVEN-FOLD WAY OF
ORTHOGONAL POLYNOMIALS

$$G_k(i, L, M, N) = {}_3F_2 \left[\begin{matrix} -k, -i, k-M-1 \\ -L, -N \end{matrix} ; 1 \right]$$

GONIN $i \in (0, N)$

$$p = -L$$

$$q = -M+L$$

$$Q_k(i, p-1, q-1, N) = {}_3F_2 \left[\begin{matrix} -k, -i, k+p+q-1 \\ p, -N \end{matrix} ; 1 \right]$$

HAHN $i \in (0, N)$

$$N \rightarrow \infty$$

$$i \rightarrow \infty$$

$$(x/E) = (i/N)$$

$$J_k(p, p+q-1, x) = {}_2F_1 \left[\begin{matrix} -k, k+p+q-1 \\ p-1 \end{matrix} ; 1 \right]$$

JACOBI $x \in (0, E)$

$$M \rightarrow \infty \quad L \rightarrow \infty$$

$$L/M = \alpha$$

$$K_k(i, \alpha, N) = {}_2F_1 \left[\begin{matrix} -k, -i \\ -N \end{matrix} ; \alpha^{-1} \right]$$

KRAWTCHOUK $i \in (0, N)$

$$p = -N$$

$$c = \alpha/(1-\alpha)$$

$$M_k(i, p, c) = {}_2F_1 \left[\begin{matrix} -k, -i \\ p \end{matrix} ; \frac{c}{1-c} \right]$$

MEIXNER $i \in (0, \infty)$

$$i \rightarrow \infty$$

$$c \rightarrow 1$$

$$i(1-c) = \beta x$$

$$L_k^{(p-1)}(x) = \frac{(p)_k}{k!} {}_1F_1 \left[\begin{matrix} -k \\ p \end{matrix} ; x \right]$$

LAGUERRE $x \in (0, \infty)$

$$N \rightarrow \infty \quad q \rightarrow \infty$$

$$N/(N+q) = c$$

$$E \rightarrow \infty \quad q \rightarrow \infty$$

$$\beta = q/E$$

$$N \rightarrow \infty$$

$$\alpha \rightarrow 0$$

$$N\alpha = \lambda$$

$$cp = \lambda$$

$$c \rightarrow 0$$

$$p \rightarrow \infty$$

$$C_k(i, \lambda) = {}_2F_0 \left[\begin{matrix} -k, -i \\ - \end{matrix} ; -\lambda^{-1} \right]$$

CHARLIER $i \in (0, \infty)$

$$G_k(i, L, M, N) = Q_k(i, -L-1, -M+L-1, N)$$

$$Q_k(i, p-1, q-1, N) = G_k(i, -p, -p-q, N)$$

$$K_k(i, \alpha, N) = M_k(i, -N, \alpha/(1-\alpha))$$

$$M_k(i, p, c) = K_k(i, c/(1+c), -p)$$

$$(i)_p = i(i+1)\dots(i+p-1)$$