THE SEVEN-FOLD WAY OF PROBABILITY DISTRIBUTIONS

HYPERGEOMETRIC

$W(i,N,L,M) =$

$$\binom{L}{i}\binom{M-L}{N-i}\Big/\binom{M}{N}$$

GONIN $\quad i \in (0,N)$

$p = -L$
$q = -M+L$

NEG HYPERGEOMETRIC

$$W_{p,q}(i,N) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)}\,\frac{(i+1)_{p-1}(N-i+1)_{q-1}}{(N+1)_{p+q-1}}$$

HAHN $\quad i \in (0,N)$

$N \to \infty$
$i \to \infty$

$(x/E) = (i/N)$

BETA

$$w_{p,q}(x,E)\,dx = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)}\,\frac{x^{p-1}(E-x)^{q-1}}{E^{p+q-1}}\,dx$$

JACOBI $\quad x \in \langle 0,E\rangle$

$M \to \infty \quad L \to \infty$

$L/M = \alpha$

$N \to \infty \quad q \to \infty$

$N/(N+q) = c$

$E \to \infty \quad q \to \infty$

$\beta = q/E$

BINOMIAL

$W(i,N;\alpha) =$

$$\binom{N}{i}\alpha^{i}(1-\alpha)^{N-i}$$

KRAWTCHOUK $\quad i \in (0,N)$

$p = -N$
$c = \alpha/(1-\alpha)$

NEG BINOMIAL

$$W_p(i,c) = \frac{(i+1)_{p-1}}{\Gamma(p)}(1-c)^p c^i$$

MEIXNER $\quad i \in (0,\infty)$

$i \to \infty$
$c \to 1$

$i(1-c) = \beta x$

GAMMA

$w_p(x,\beta)\,dx =$

$$\frac{\beta^p x^{p-1} e^{-\beta x}}{\Gamma(\rho)}\,dx$$

LAGUERRE $\quad x \in \langle 0,\infty\rangle$

$N \to \infty$
$\alpha \to 0$

$N\alpha = \lambda$

POISSON

$W(i,\lambda) =$

$$\frac{\lambda^i}{i!}e^{-\lambda}$$

CHARLIER $\quad i \in (0,\infty)$

$cp = \lambda$

$c \to 0$
$p \to \infty$

$$W_p(i,c) = W(i,-p,c/(1-c))$$

$$W(i,N,\alpha) = W_{-N}(i,\alpha/(\alpha-1))$$

$$W_{p,q}(i,N) = W(i,N,-p,-p-q)$$

$$W(i,N,M,L) = W_{-L,-M+L}(i,N)$$

$$(i)_p = i(i+1)\ldots(i+p-1)$$

M. HOARE 5.80