Appendix 8: Pictures of prime numbers and ideals for real fields of class number 3

The pictures show the quadratic character and a picture of prime numbers, units and two mutually conjugate classes of non-principal prime ideals, one class red, and the other class green for some real quadratic fields of class number 3, namely

the fields of discriminant congruent 0 modulo 4:

\[ Q(\sqrt{79}), Q(\sqrt{142}), Q(\sqrt{223}), Q(\sqrt{254}), Q(\sqrt{326}), Q(\sqrt{359}) \]

and the fields of discriminant congruent 1 modulo 4:

\[ Q(\sqrt{229}), Q(\sqrt{257}), Q(\sqrt{321}). \]

The pictures display the prime numbers, which generate the principal prime ideals, but not those irreducible numbers which are not prime. Moreover, the non-principal prime ideals are displayed as follows. The non-principal ideals are obtained by dividing principal ideals by a certain non-principal prime ideal, \( I \), or its conjugate, where

\[ I := \{ \text{norm}, \zeta \}, \zeta := \text{shift} + (d \mod 4 + \sqrt{d}) / 2, \]

i.e. \( I \) is generated by ‘norm’ being its norm, and the integer \( \zeta \) of \( \mathbb{Q}(\sqrt{r}) \).

In the picture, the non-principal prime ideals then are represented by those numbers whose norm is equal to a prime norm times the norm of \( I \). This norm of \( I \) and shift are mentioned at the top of the picture, shift being needed to distinguish between the two mutually conjugate classes of non-principal ideals.