Thermodynamics of black holes

Bekenstein 1973: Black hole has entropy $S_{BH} = \frac{A}{4L_P^2}$
(L_P is Planck length$=\sqrt{\hbar G/c^3}$)

Hawking 1975: Black holes have temperature
$T_H = \frac{\hbar G}{2\pi c^3}\kappa$ (κ=surface tension)

The four laws of black hole dynamics (Bardeen, Carter, Hawking, 1975)

0: T_H is constant at the horizon

1: $\mathrm{d}U = T_H \mathrm{d}S_{BH} + \mathrm{d}W$

2: Total entropy $S = S_{BH} + S_m$ cannot decrease

3: Maximally charged (rotating) black holes,

with $T_H = 0$, unattainable

Are these the laws of black hole thermodynamics?

Thermal bath = cosmic microwave background

First law: System=black hole +Gedanken sphere

Try: $đQ = T_H \, \mathrm{d}S_{BH} + T_{mcb} \, \underbrace{\mathrm{d}S_m^{\text{Gedanken sphere}}}_{=0}$

$\mathrm{d}U = đQ + đW = T_H \, \mathrm{d}S_{BH} + đW$ QED

Second law Clausius:

$đQ \leq T_{cmb} \mathrm{d}[S_{BH} + S_m^{\text{Gedanken sphere}}]$?

Yes, because there is also absorption of CMB

$\begin{cases} \text{small BH:large } T_H \text{, more evaporation} \\ \text{large BH:small } T_H \text{, more absorption} \end{cases}$

$\rightarrow$ heat goes from high T to low T QED

Third law: System= Universe with BH in it.

$\rightarrow S = 0$ if $T_{cmb} = 0$, not $T_H = 0$.

$T_{cmb} < t^{-1/3} \rightarrow$ all BH's finally evaporate

$\rightarrow S = 0$ at $T_{cmb} = 0$ QED

(deviates from third law of dynamics)