Bath generated work extraction in two-level systems

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Hamiltonian for spin $\frac{1}{2}$ in bosonic bath

$$\mathcal{H} = \mathcal{H}(\Delta) = \mathcal{H}_S + \mathcal{H}_B + \mathcal{H}_I,$$

$$\mathcal{H}_S = \frac{\varepsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x,$$

$$\mathcal{H}_B = \sum_k \hbar\omega_k \hat{a}_k^\dagger \hat{a}_k, \quad \mathcal{H}_I = \frac{1}{2}\sum_k g_k(\hat{a}_k^\dagger + \hat{a}_k)\hat{\sigma}_z.$$

$\hat{\sigma}_z$, $\hat{\sigma}_x$: Pauli-matrices (2×2)

$\hat{a}_k^\dagger$, $\hat{a}_k$: creation/annihilation operators for phonons/photons.

$\varepsilon = \bar{g}\mu_B H$, $H =$ field in z-direction

Δ field in x-direction. If $\Delta = 0$: exactly solvable.

$\Delta = 0$ except in pulse.

Quasi-Ohmic bath with Debye cutoff frequency Γ

$$J(\omega) = \sum_k \frac{g_k^2}{\hbar\omega_k}\delta(\omega_k - \omega) = \frac{g\,\hbar}{\pi} e^{-\omega/\Gamma}$$

g dimensionless coupling, $\frac{1}{g}$ 'quality factor'

Separated initial state: $\rho(0) = \rho_S(0) \otimes \frac{\exp(-\beta\mathcal{H}_B)}{Z_B}$

Decay of transverse components

$$\langle\hat{\sigma}_x(t)\rangle = [\cos\omega_0 t\,\langle\hat{\sigma}_x(0)\rangle - \sin\omega_0 t\,\langle\hat{\sigma}_y(0)\rangle]\; e^{-\xi(t)}, \qquad \omega_0 = \frac{\varepsilon}{\hbar}$$

$$\langle\hat{\sigma}_y(t)\rangle = [\sin\omega_0 t\,\langle\hat{\sigma}_x(0)\rangle + \cos\omega_0 t\,\langle\hat{\sigma}_y(0)\rangle]\; e^{-\xi(t)}$$

For $t \gg 1/\Gamma$: the T_2-timescale (exponential decay)

$$\xi(t) \approx \frac{t}{T_2}, \qquad T_2 = \frac{1}{g}\,\frac{\hbar}{T}$$

picoseconds $\leq T_2 \leq$ minutes $\rightarrow 1 \geq g \geq 10^{-14}$

Entropy increases (second law during relaxation),

$S_{\mathrm{vN}} = -\frac{1+|\langle\vec{\sigma}\rangle|}{2}\ln\frac{1+|\langle\vec{\sigma}\rangle|}{2} - \frac{1-|\langle\vec{\sigma}\rangle|}{2}\ln\frac{1-|\langle\vec{\sigma}\rangle|}{2}$.

because it only depends on

$|\langle\vec{\hat{\sigma}}(t)\rangle| = \sqrt{\langle\hat{\sigma}_x(t)\rangle^2 + \langle\hat{\sigma}_y(t)\rangle^2 + \langle\hat{\sigma}_z(t)\rangle^2}$

Pulse = fast rotation around x-axis, angle θ

$$U_1^{-1}\,\hat{\sigma}_y\,U_1 = \hat{\sigma}_z\sin\theta + \hat{\sigma}_y\cos\theta,$$
$$U_1^{-1}\,\hat{\sigma}_z\,U_1 = \hat{\sigma}_z\cos\theta - \hat{\sigma}_y\sin\theta$$

For $\theta = -\frac{1}{2}\pi$:

$$W_1 = \frac{g\ \hbar\Gamma}{2\pi} - \left[\frac{\varepsilon}{2}\sin\omega_0 t + \frac{gT}{2}\cos\omega_0 t\right]\tanh\frac{\beta\varepsilon}{2}\ e^{-t/T_2}$$

Heat taken from bath:

$$\Delta Q = \frac{g}{2}\left[-\frac{\hbar\Gamma}{\pi} + T\cos\omega_0 t\ \tanh\frac{\beta\varepsilon}{2}\,e^{-t/T_2}\right]$$

Violation of Clausius inequality in pulse

During pulse: $\Delta S = 0$, since pulse = rotation

$\Delta Q > 0$ occurs when t not too large, T intermediate.

Two pulses

If many weakly interacting spins, each in different field: inhomogeneous broadening of $\omega_0 = \frac{\varepsilon}{\hbar}$: line

$$p(\omega_0) = \frac{2}{\pi} \frac{[T_2^*]^{-1}}{(\omega_0 - \bar{\omega}_0)^2 + [T_2^*]^{-2}}$$

average $\bar{\omega}_0$, inverse width $T_2^* \ll T_2$.

- Single pulse: only loss of work

- First $-\frac{1}{2}\pi$ pulse around the x-axis at time t_1 and a second $\frac{1}{2}\pi$ pulse at time $t_2 = t_1 + \Delta t$

Total work W

$$\begin{aligned} W &= \frac{g\ \hbar\Gamma}{\pi} - \frac{\hbar}{4} e^{-t_2/T_2} e^{-|\Delta t|/T_2^*} \tanh \frac{\beta\ \hbar\bar{\omega}_0}{2} \{\bar{\omega}_0 \sin \bar{\omega}_0 \Delta t \\ &+ [\frac{1}{T_2} - \frac{\mathrm{sg}(\Delta t)}{T_2^*}(1 + \frac{\beta\ \hbar\bar{\omega}_0}{\sinh \beta\ \hbar\bar{\omega}_0})] \cos \bar{\omega}_0 \Delta t \} \end{aligned}$$

For Δt near $2\pi n/\bar{\omega}_0$ such that odd terms cancel, this again exhibits work extracted solely from the bath.

Feasibility

1) Many quantum system act as two-level systems;

2) Detection is relatively easy, only $\langle\vec{\sigma}(t)\rangle$ needed;

3) The harmonic oscillator bath is universal;

4) Work and heat measured in NMR 35 years ago;

5) many-spin measurements allowed;

6) quantum computing: quite long $\mathcal{T}_2$ times, admit external variations on times smaller than $\mathcal{T}_2$;

(i) for atoms in optical traps $\mathcal{T}_2 \sim 1$s, $1/\Gamma \sim 10^{-8}$;
(ii) electron injected in semiconductor $\mathcal{T}_2 \sim 1\,\mu$s;
(iii) exciton created in a quantum dot $\mathcal{T}_2 \sim 10^{-9}$s.

Conclusion

- Clausius inequality and Thomson formulation violated, (pulse = cycle in transverse field Δ).

- The mechanism exploits the quantum nature of spins: transverse components.

- 'Classical' pulses with $\theta = \pm\pi$ do not lead to violations, (no transversal components created).

- no work extraction from equilibrium initial state