Quantum Limits to the Second law

Armen E. Allahverdyan

Yerevan/Amsterdam/Saclay

Theo M. Nieuwenhuizen

Institute for Theoretical Physics, University of Amsterdam

- Extraction of work from a single thermal bath in the quantum regime, PRL 85 (2000) 1799

- Breakdown of the Landauer bound for information erasure in the quantum regime PRE, in press

- Statistical thermodynamics of quantum Brownian motion: Birth of perpetuum mobile of the second kind, in preparation

Setup

- Quantum particle with harmonic oscillator bath
- Harmonic force of quantum particle
  $\rightarrow$ exactly solvable system
- Thermodynamic interpretation
- Energy relaxation
- Entropy production
- Work extraction cycles
- Perpetuum mobile and Maxwell's demon
- Summary

For Ubbo only:

Eddington 1935:

The law of entropy increase - the second law of thermodynamics - holds, I think, the supreme position among the laws of Nature.

If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations - then so much the worse for Maxwell's equations.

If it is found to be contradicted by observation, well, these experimentalists do bungle things sometimes.

But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but collaps in deepest humiliation.

Quantum brownian motion

$$\mathcal{H} = \frac{p^2}{2m} + \frac{1}{2}ax^2 + \sum_i \left[\frac{p_i^2}{2m_i} + \frac{m_i\omega_i^2}{2}(x_i - \frac{c_i\,x}{m_i\omega_i^2})^2 \right]$$

particle+oscillator bath+interaction $x_i x + x^2$

Exactly solvable problem

"Caldeira-Leggett" model

→ P. Ullersma (Utrecht), 4 papers in Physica 1966

Examples of quantum brownian motion:

- fluctuation effects in Josephson junctions
- low-temperature quantum transport
- quantum-optical systems

- $\omega_i = i\,\Delta$: level spacing $\Delta \to 0$:

- Spectral density: $J(\omega) = \sum_i \frac{\pi c_i^2}{2m_i\omega_i}\delta(\omega_i - \omega)$
- Ohmic: $J(\omega) = \gamma\omega$
- quasi-Ohmic: $J(\omega) = \frac{\Gamma^2}{\omega^2+\Gamma^2}\gamma\omega$

(Drude-Ullersma)

Equivalent Langevin equation (operator-valued)

$$\underbrace{\dot{p}}_{\text{acceleration}} = \underbrace{-ax}_{\text{force}} - \underbrace{\frac{\gamma\Gamma}{m}\int_0^t \mathrm{d}t' e^{-\Gamma(t-t')}p(t')}_{\text{friction}} + \underbrace{\eta(t)}_{\text{noise}}$$

$$\underbrace{\frac{1}{2}\langle\{\eta(t),\eta(0)\}\rangle}_{\text{noise}-\text{correlator}} = K(t) = \frac{\hbar\gamma}{2\pi}\int \mathrm{d}\omega \frac{\omega\coth(\hbar\omega\beta/2)e^{i\omega t}}{1+(\omega/\Gamma)^2}$$

- anticorrelation: $K(t) < 0$ for $t \gg 1/\Gamma$
- timescale of friction $1/\Gamma$; of noise: $\max(\hbar/T, 1/\Gamma)$

$\to$ non-Gibbsian behavior for $T < \hbar\Gamma$

At $t = 0$:

- Density matrix of total system is Gibbsian
- Spring constant $a_0 \to a = (1+\alpha)a_0 \quad (|\alpha| \ll 1)$
- Work done in switching: $\mathcal{W}_0 = \frac{1}{2}(a - a_0)\langle x^2 \rangle_0$
- $t > 0$: Subsystem (Brownian particle) relaxes

Standard expectation:

$$\text{state of particle} \underbrace{\to}_{t\to\infty} \rho = \underbrace{\frac{\exp[-\frac{1}{T}(\frac{p^2}{2m} + V(x))]}{Z}}_{\text{Gibbs distribution}}$$

$$\rho = \frac{1}{Z}\sum_n e^{-E_n/T}|n\rangle\langle n| \underbrace{\to}_{T\to 0} \underbrace{|0\rangle\langle 0|}_{\text{pure state}}$$

Impossible if $\gamma \neq 0$. Reason: quantum entanglement. An interacting quantum subsystem cannot be in a pure state

$$\begin{pmatrix} \mathcal{H}_S & \mathcal{H}_I \\ \mathcal{H}_I & \mathcal{H}_B \end{pmatrix}$$

Stationary distribution (Haake, Reibold; Unruh, Zurek)

$$W_{st}(x,p) = \frac{1}{Z_x}\,\exp(-\frac{ax^2}{2T_x}) \,\times\, \frac{1}{Z_p}\,\exp(-\frac{p^2}{2mT_p})$$

It is non-Gibbsian since $T_x \neq T_p$

$\rightarrow$ equilibrium thermodynamics endangered

$$T_x = T + \frac{\hbar a}{\pi m}\left\{\frac{\omega_1 - \Gamma}{(\omega_2-\omega_1)(\omega_3-\omega_1)}\psi\left(\frac{\beta\hbar\omega_1}{2\pi}\right) + \text{cyclic}\right\}$$

$$T_p = T_x + \frac{\hbar\gamma\Gamma}{\pi m}\left\{\frac{\omega_1}{(\omega_2-\omega_1)(\omega_3-\omega_1)}\psi\left(\frac{\beta\hbar\omega_1}{2\pi}\right) + \text{cyclic}\right\}$$

At $T=0$: $T_p > T_x > 0$

Thermodynamic description for adiabatic changes

1) Violation of Clausius inequality $đQ \leq T \; \mathrm{d}S$

$$đQ(T \to 0) = \frac{\hbar\gamma}{2\pi m^2} \mathrm{d}m \neq 0, \quad \textit{comes from cloud}$$

2) One can adsorb heat, $đQ > 0$, and become more localized, $\mathrm{d}S < 0$.

3) Breakdown of Landauer bound for erasure of one bit of information $|đQ| \geq T \ln 2$

4) Generalized Clausius inequality:

$$\boxed{đ\,Q = T_x \; \mathrm{d}S_x + T_p \; \mathrm{d}S_p \qquad \text{as for glasses \& black holes}}$$

Effective temperatures T_x, T_p,

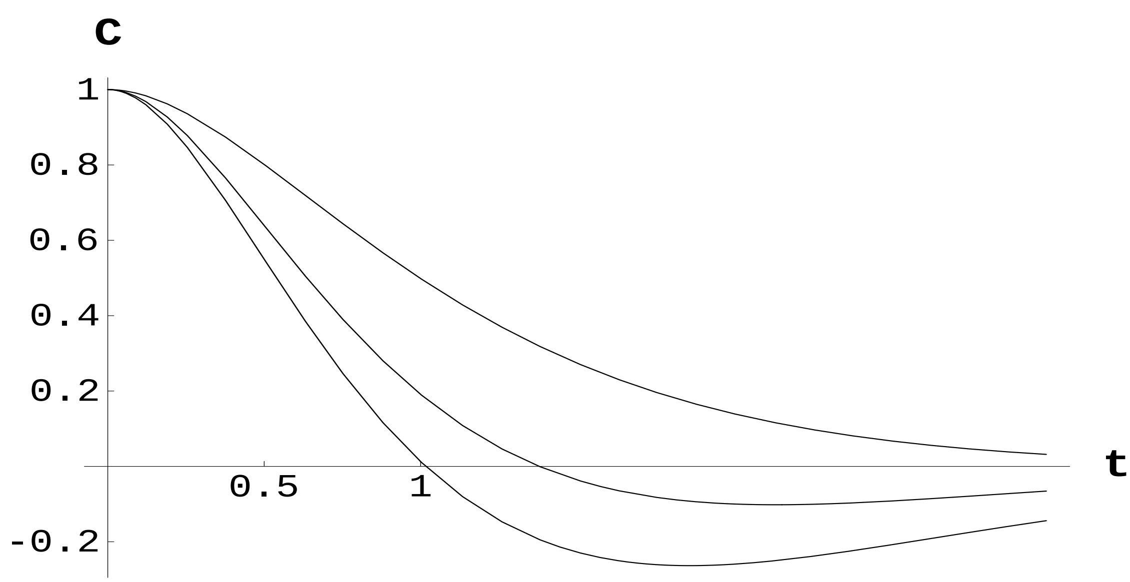
Boltzmann entropies of x and p - parts of Wigner fion

$$S_x = -\int \mathrm{d}x \, W_x(x) \ln W_x(x), \qquad S_p = -\int \mathrm{d}p \, W_p(p) \ln W_p(p)$$

Relaxation of the energy

$$U(t) = \frac{1}{2}T_x + \frac{1}{2}T_p + \alpha\ C(t), \qquad \alpha = \frac{a - a_0}{a}$$

- Strong damping: $(\gamma \gg \sqrt{am})$



Upper: large T;

Middle: moderate T

Lower: $T = 0$; the integral vanishes

Production of Boltzmann entropy

$$S_B = -\int \mathrm{d}x\mathrm{d}p W(x,p,t)\, \ln\, W(x,p,t)$$

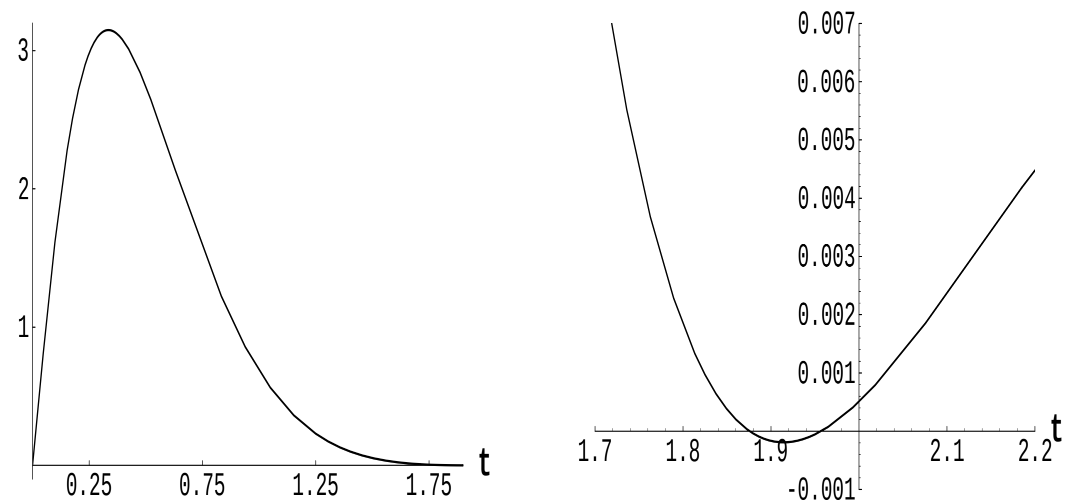
change = flow +production: $\frac{\mathrm{d}S_B}{\mathrm{d}t} = \frac{\mathrm{d}_e S_B}{\mathrm{d}t} + \frac{\mathrm{d}_i S_B}{\mathrm{d}t}$

entropy flow: $\frac{\mathrm{d}_e S_B}{\mathrm{d}t} = \frac{\partial_t \langle ax^2 \rangle /2}{D_x(t)} + \frac{\partial_t \langle p^2 \rangle /2m}{D_p(t)}$

$D_x \neq D_p \neq T$: Diffusion coefficients from Fokker-Planck equation for Wigner function

Entropy production: $\frac{\mathrm{d}_i S_B}{\mathrm{d}t}$ quadratic in α

- Strong damping, $T = 0$



Entropy production as function of time

Non-adiabatic Energy dispersion:

$$đ\mathcal{W} = đ\mathcal{W}_{\mathrm{adiabatic}} + đ\Pi$$

Slow change of $a(t) = [1 + \alpha(t)]a$ at small T

$\frac{đ\Pi}{\mathrm{d}t} = T^2\,\dot{\alpha}^2 + \dot{\alpha}\ddot{\alpha} - \dot{\alpha}\partial_t^3\alpha$

- Full change: Thomson formulation of second law

$\Delta\Pi = \int_{-\infty}^{\infty}\mathrm{d}t\,\frac{đ\Pi}{\mathrm{d}t} = \int \mathrm{d}t[T^2\,\dot{\alpha}^2 + \ddot{\alpha}^2] \geq 0$

- Fixed time: $\dot{\alpha}\ddot{\alpha}$ term dominates: $\frac{đ\Pi}{\mathrm{d}t} < 0$ possible

At low T change may contain cycle(s) in which energy is extracted (but total dispersion is positive)

"Perpetuum mobile of second kind"

Maximal number of cycles $\sim \frac{1}{T}$

Optimal extraction in one cycle (else additional dispersion)

What goes wrong with standard thermodynamics ?

- The interaction energy is non-zero
- Non-additivity of quantum entropy
  $S_{System+Bath} \neq S_{System} + S_{Bath}$

Conclusions

- Harmonic quantum particle coupled to harmonic bath
- Damping $\approx$ instantaneous but noise $\neq$ white
- Second law is violated:
  Clausius inequality broken at all T;
  Landauer bound for erasure of information violated
- Work extraction if low T and non-equilibrium:
  "Perpetuum mobile of second kind"
- Reason: damping γ is not small $\rightarrow$ entanglement between system and bath
- Unmasking Maxwell's demon: quantum entanglement