Quantum Brownian motion

and its conflict with the second law

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Extraction of work from a single thermal bath in the quantum regime, Phys. Rev. Lett. 85 (2000) 1799

Breakdown of the Landauer bound for information erasure in the quantum regime,
Phys. Rev. E 64, 056117 (2001)

Statistical thermodynamics of quantum Brownian motion: Construction of perpetuum mobile of the second kind, Phys. Rev. E, in press

On testing the violation of the Clausius inequality in nanoscale electric circuits
Phys. Rev. B, in press

Setup

- Harmonic quantum particle
  in harmonic oscillator bath $\rightarrow$ exactly solvable
- Thermodynamic interpretation
- Energy relaxation
- Entropy production
- Energy dispersion
- Work extraction cycles
- Summary

Quantum Brownian motion

$$\mathcal{H}_{tot} = \frac{p^2}{2m} + \frac{1}{2}ax^2 + \sum_i \left[\frac{p_i^2}{2m_i} + \frac{m_i\omega_i^2}{2}(x_i - \frac{c_i\, x}{m_i\omega_i^2})^2 \right]$$

particle+oscillator bath+interaction $x_i x + x^2$

- $\omega_i = i\,\Delta$: level spacing $\Delta \to 0$:

- Spectral density: $J(\omega) = \sum_i \frac{\pi c_i^2}{2m_i\omega_i}\delta(\omega_i - \omega)$
- Ohmic: $J(\omega) = \gamma\omega$
- quasi-Ohmic: $J(\omega) = \frac{\Gamma^2}{\omega^2+\Gamma^2}\gamma\omega$ (Drude-Ullersma)

"Caldeira-Leggett" model; Ullersma 1966

Applications

- fluctuation effects in Josephson junctions
- low-temperature quantum transport
- quantum-optical systems

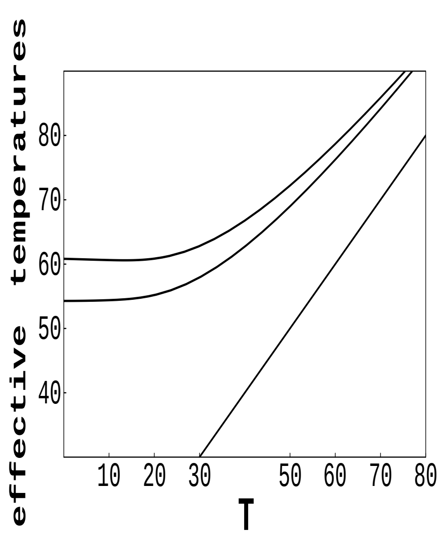
At $t = 0$:

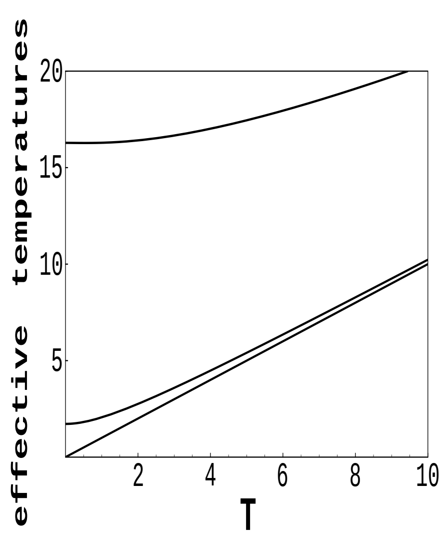
- Density matrix of total system is Gibbsian at T
- Spring constant $a_0 \to a = (1+\alpha)a_0 \quad (|\alpha| \ll 1)$
- Work done in switching: $\mathcal{W}_0 = \frac{1}{2}(a - a_0)\langle x^2 \rangle_0$
- $t > 0$: Subsystem (Brownian particle) relaxes

Stationary distribution: Wigner function
(Haake, Reibold 1985; Unruh, Zurek 1989):

$$W_{st} = W_p(p) \times W_x(x) = \frac{1}{Z_p} \exp(-\frac{p^2}{2mT_p}) \times \frac{1}{Z_x} \exp(-\frac{ax^2}{2T_x})$$

It is non-Gibbsian since $T_x \neq T_p$ (both exceeding T)
$\to$ equilibrium thermodynamics endangered





Thermodynamic description for adiabatic changes

$$U = \langle \mathcal{H} \rangle = \int \mathrm{d}p\mathrm{d}x\mathcal{H}(p,x)W(p,x) \equiv \int \mathcal{H}W$$

0) First law: $\mathrm{d}U = \int \mathcal{H}\mathrm{d}W + \int W\mathrm{d}\mathcal{H} \equiv đQ + đ\mathcal{W}$

1) Violation of Clausius inequality $đQ \leq T\ \mathrm{d}S$

$$đQ(T \to 0) = \frac{\hbar\gamma}{2\pi m^2}\mathrm{d}m \neq 0, \quad \textit{comes from cloud}$$

2) One can adsorb heat, $đQ > 0$, and become more localized, $\mathrm{d}S < 0$.
Breakdown of Landauer bound for erasure of one bit of information $|đQ| \geq T \ln 2$

3) Generalized Clausius inequality:

$$đ\,Q = T_p\ \mathrm{d}S_p + T_x\ \mathrm{d}S_x \qquad \text{as for glasses \& black holes}$$

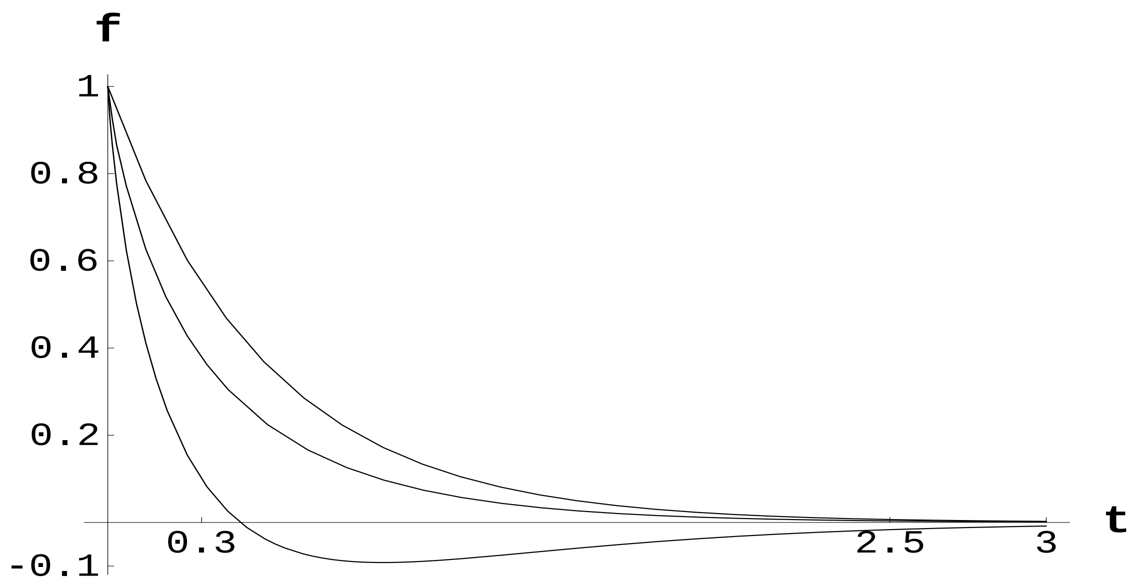
Boltzmann entropies of p and x - parts of Wigner fion

$$S_p = -\int \mathrm{d}p\,W_p(p)\ln W_p(p), \qquad S_x = -\int \mathrm{d}x\,W_x(x)\ln W_x(x)$$

Relaxation of the energy

$U(t) = \frac{1}{2} T_x + \frac{1}{2} T_p + \alpha \; C(t), \qquad \alpha = \frac{a - a_0}{a}$

- Strong damping: $(\gamma \gg \sqrt{am})$



Upper: large T;

Middle: moderate T

Lower: $T = 0$; the integral vanishes

Production of Boltzmann entropy

$$S_B = -\int \mathrm{d}x\mathrm{d}p W(x,p,t)\, \ln[\hbar\, W(x,p,t)]$$

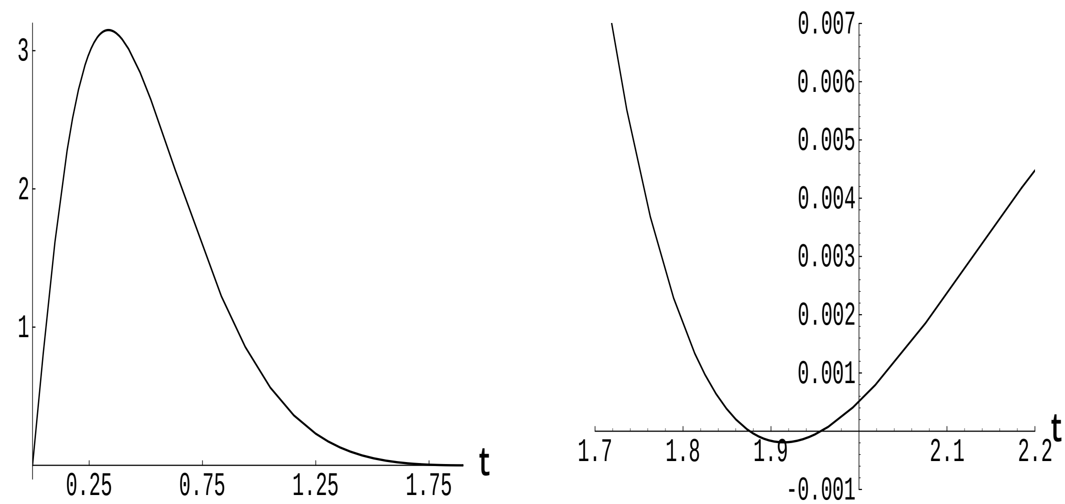
change = flow +production: $\frac{\mathrm{d}S_B}{\mathrm{d}t} = \frac{\mathrm{d}_e S_B}{\mathrm{d}t} + \frac{\mathrm{d}_i S_B}{\mathrm{d}t}$

entropy flow: $\frac{\mathrm{d}_e S_B}{\mathrm{d}t} = \frac{\partial_t \langle a x^2\rangle/2}{D_x(t)} + \frac{\partial_t \langle p^2\rangle/2m}{D_p(t)}$

$D_x \neq D_p \neq T$: Diffusion coefficients from Fokker-Planck equation for Wigner function

Rate of entropy production: $\frac{\mathrm{d}_i S_B}{\mathrm{d}t}$ quadratic in α

- Strong damping, $T = 0$



Entropy production as function of time

Rate of energy dispersion:

$$đ\mathcal{W} = đ\mathcal{W}_{\text{adiabatic}} + đ\Pi$$

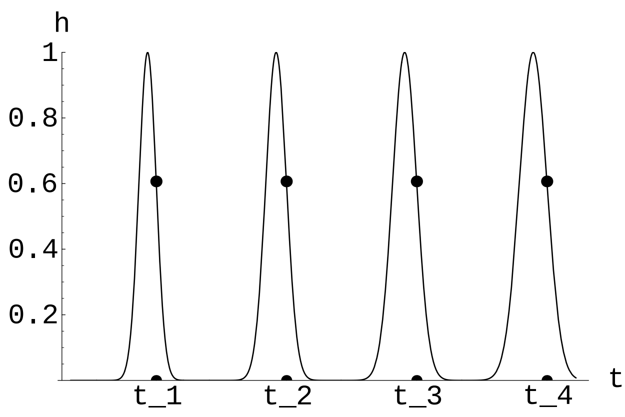
Slow change of $a(t) = [1 + \alpha(t)]a$ at small T

$$\frac{đ\Pi}{dt} = T^2\,\dot{\alpha}^2 + \dot{\alpha}\ddot{\alpha} - \dot{\alpha}\partial_t^3\alpha$$

- Full change: Thomson formulation of second law

$$\Delta\Pi = \int_{-\infty}^{\infty} dt\,\frac{đ\Pi}{dt} = \int dt[T^2\,\dot{\alpha}^2 + \ddot{\alpha}^2] \geq 0$$

- Fixed time: $\dot{\alpha}\ddot{\alpha}$ term dominates: $\frac{đ\Pi}{dt} < 0$ possible



Many $\sim \frac{1}{T}$ work extraction cycles possible.

What goes wrong with thermodynamics ?

- The interaction energy is non-zero: $U_{int} \sim \gamma$
  Thermodynamics needs not apply when
  $T < T_* \sim \gamma$
- But rate of entropy production negative (and energy relaxation non-monotonous) for $T \gg T_*$
- Rate of energy dispersion (of total system) can be negative when $T < T_*$
- Out of equilibrium $\sim \frac{1}{T}$ cycles possible in which energy is extracted.
  "Perpetuum mobile of second kind"
  Optimal extraction if one cycle