Thomson's formulation of the second law

an exact theorem and limits of its validity

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A mathematical theorem as the basis for the second law: Thomson's formulation applied to equilibrium, Physica A **305**, 542 (2002)

Extracting work from a macroscopic thermal bath via a mesoscopic work source, submitted to Phys. Rev. E (2002)

Mesoscopic limitations of Thomson's formulation of the second law, preprint (July, 2002)

Setup

- Thomson's law as a theorem for macroscopic sources.
- Mesoscopic sources limit the validity of Thomson's law.
- Conclusion.

The setup of Thomson's law

- system S (engine)
- equilibrium thermal bath B at temperature T
- work source W

General (heuristic) formulation of the Thomson's law:

No work can be extracted from B to W by means of a cyclic process made by S.

Recall: work $\mathcal{W}$ on S = minus change in energy of W.

For macroscopic sources:

1) interaction between S and W = variation with time of a parameter ξ of S. Examples: piston for a gas; magnetic field for spins, etc.

2) the equilibrium initial state of S: Gibbs distribution with its Hamiltonian H_S and temperature $T > 0$.

3) a cyclic process at time τ is achieved by $\xi(0) = \xi(\tau)$.

Theorem: under conditions 1),2),3) one always has:
$\mathcal{W} \geq 0$: Thomson's law for macroscopic sources.

Mesoscopic case: setup for work-extraction from B

- Total Hamiltonian:

$$H = H_S + H_W + g(t)H_I$$

  H_I: interaction, $g(t)$: coupling constant

- Cyclic process in a time-interval $(0, \tau)$:

  $g(0) = 0$, $g(\tau) = 0$, $g = \text{const} \neq 0$ for $0 < t < \tau$;

  Total energy conserved when switching g on and off, iff $\langle H_I \rangle_0 = \langle H_I \rangle_\tau = 0$.

- switching fast: influence of B negligible for $t < \tau$.

- S starts from equilibrium as always, and returns finally to equilibrium for large t.

- W starts from a macroscopic (deterministic) state.

Main results:

- Work-extraction is possible

- Infinite amount of work-extracting cycles with a finite work per cycle is possible.

Illustrations: Exactly solvable models.

Jaynes-Cummings model: oscillator interacting with a spin. (Applications: quantum optics and electronics, NMR-physics).

$$\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a} + \frac{1}{2}\,\hbar\omega\hat{\sigma}_z + \hbar g(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger),$$

$\hat{a}$, $\hat{a}^\dagger$ are annihilation/ creation operators,
$\hat{\sigma}_x$, $\hat{\sigma}_y$, $\hat{\sigma}_z$, $\hat{\sigma}_{+,-} = (\hat{\sigma}_x \pm i\hat{\sigma}_y)/2$ are Pauli matrices.

• Thomson's law only recovered for low T and/or for high intensities of the mode.

Two interacting oscillators:

$$H_\mathrm{S} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\omega^2x^2, \quad H_\mathrm{W} = \frac{1}{2}M\dot{y}^2 + \frac{1}{2}M\Omega^2y^2,$$
$$H_I = -g\,xy$$

• T large enough: many work extraction cycles possible

• if source emptied, fixed work per cycle, infinitely many cycles: perpetuum mobile, cools macroscopic bath.

Conclusion

- Thomson's formulation valid as a theorem for macroscopic sources.
- For mesoscopic sources of work it can be violated: Work can be extracted from macroscopic thermal bath in equilibrium.
- Infinite amount of work-extraction cycles with a finite energy per cycle are possible.