Social Software for Coalition Formation

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Structure:
1. The model of coalition formation.
2. Applying the MacBeth technique.
3. Applying RelVieW.
4. Applying Graph Theory.

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1. The Model of Coalition Formation

Let $N = \{1, \ldots, n\}$ be the set of *parties* or *agents*. A subset of N is called a *coalition*. Let W be the set of *winning coalitions*.

Let $C = \{c_0, c_1, \ldots, c_m\}$ be the set of (independent) *policy issues* or *criteria*, where c_0 denotes the issue of the coalition; c_1 concerns, for instance, health care, c_2 education, etc. Let P be the set of relevant *policies* $p = (p_1, \ldots, p_m)$, p_j being the policy on issue j.

A *government* $g = (S, p)$ is a pair consisting of a coalition S and a policy p.

The set of all governments is

$$G := \{ \ (S, p) \mid S \in W \wedge p \in P \ \}$$

We assume an *acceptability relation* $A : N \leftrightarrow G$:

$$A_{i,g} \iff \text{party } i \text{ accepts government } g$$

i.e., with respect to every criterium the utility of g for i is greater than or equal to 0.

A government $g = (S, p)$ is said to be *feasible* if both S and p are acceptable to each party belonging to S.
The set of all feasible governments is

$$G^* := \{\, g \in G \mid g \text{ is feasible} \,\}$$

In the paper we describe a procedure for a winning coalition to *reach consensus* on a policy in order to form a feasible government.

A *decision maker* is a party involved in at least one feasible government.
Let DM be the set of all decision makers.

Each feasible government is evaluated by each decision maker with respect to the given criteria. Different parties may give different weights to the different criteria.

Let $\alpha_i(c) \in [0, 1]$ be the weight that decision maker i gives to issue c. We assume

$$\forall i \in DM \ [\sum_{c \in C} \alpha_i(c) = 1]$$

Let $u_i(c,g) \in \mathbb{R}$ denote the utility of government g with respect to criterion c for party i.

Question: How to determine $\alpha_i(c)$ and $u_i(c,g)$?
Answer: Using the MacBeth software.

The utility $U_i(g)$ of government g for party i is defined by:

$$U_i(g) = \alpha_i(c_0)u_i(c_0,g) + \ldots + \alpha_i(c_m)u_i(c_m,g)$$

Example: $N = \{1,2,3\}$, $C = \{c_0, c_1, c_2\}$. Suppose there are 4 feasible governments $g_1, \ldots, g_4$. Let $(\alpha_1(c_0), \alpha_1(c_1), \alpha_1(c_2)) = (0.6,\ 0.3,\ 0.1)$ be the weights that party 1 gives to the three different issues.

$$\text{Suppose } (u_1(c,g)_{c\in C, g\in G} = \begin{array}{c} \begin{matrix} g_1 & g_2 & g_3 & g_4 \end{matrix} \\ \begin{pmatrix} 80 & 90 & 70 & 60 \\ 20 & 40 & 50 & 70 \\ 10 & 70 & 30 & 80 \end{pmatrix} \end{array}$$

Then

$(U_1(g))_{g \in G} = (U_1(g_1), U_1(g_2), U_1(g_3), U_1(g_4)) =$

$$(0.6, 0.3, 0.1) \begin{pmatrix} 80 & 90 & 70 & 60 \\ 20 & 40 & 50 & 70 \\ 10 & 70 & 30 & 80 \end{pmatrix} = (55, 73, 60, 65)$$

This gives the utilities for party 1 of the governments $g_1, \ldots, g_4$.

Let g and h be feasible governments.
$h = (S, p)$ **dominates** g, denoted by $h \succ g$, iff

$$\forall i \in S[U_i(h) \geq U_i(g)] \wedge \exists i \in S[U_i(h) > U_i(g)].$$

g is **stable** := there is no feasible government dominating g, i.e., $\neg \exists h \in G^*[h \succ g]$.

There may be many feasible governments!

Question: How to calculate the stable governments ?
Answer: Using the RelVieW software.

There may be no stable government!
Question: What to do in this case ?
Answer: Use Graph theory to break cycles.

2. Applying the MacBeth software

to determine the utilities $u_i(c, g)$ of governments g wrt a given criterion c for party i.

For every criterion c, each party i is asked to specify two particular references:
- $neutral_i^c$: a for party i neutral government with respect to criterion c
- $good_i^c$: a for party i good government with respect to criterion c

Let $G_i^c = G^* \cup \{neutral_i^c, good_i^c\}$
For each $c \in C$, each party $i \in DM$ is asked to judge verbally the difference of attractiveness between each two governments $g, h \in G_i^c$ with respect to c. When judging, a party has to choose one of the following categories:
- D_0: *no* difference of attractiveness
- D_1: *very weak* difference of attractiveness
- D_2: *weak* difference of attractiveness
- D_3: *moderate* difference of attractiveness
- D_4: *strong* difference of attractiveness
- D_5: *very strong* difference of attractiveness
- D_6: *extreme* difference of attractiveness

For every criterion c, each party i orders all governments from the best one ($g^c_{i,1}$) to the worst one ($g^c_{i,k}$) with respect to criterion c and next fills a matrix as below.

Matrix of judgements of difference of attractiveness for i between governments with respect to criterion c:

	$g^c_{i,1}$	$g^c_{i,2}$	$\ldots$	$g^c_{i,k-1}$	$g^c_{i,k}$
$g^c_{i,1}$	**no**	weak	$\ldots$	very strong	extreme
$g^c_{i,2}$		**no**	$\ldots$	strong	very strong
$\ldots$			**no**	$\ldots$	$\ldots$
$g^c_{i,k-1}$				**no**	moderate
$g^c_{i,k}$					**no**

If the matrix is inconsistent, the MacBeth software signals this. From a consistent matrix, the MacBeth software determines (by linear programming) a basic scale: values $u_i(c, g)$ of governments g with respect to criterion c, taking into account the following rules:

$u_i(c, g) > u_i(c, h) \Leftrightarrow$

g is more attractive to i wrt c than h (1)

If $(g, g') \in D_k$ and $(h, h') \in D_{k'}$ and $k > k'$, then

$$u_i(c, g) - u_i(c, g') > u_i(c, h) - u_i(c, h') \qquad (2)$$

By applying a linear transformation, we may assume that

$u_i(neutral_i^c) = 0$ and $u_i(good_i^c) = 100$.

By presenting the MacBeth scale graphically as a kind of thermometer, the party still has the possibility to adapt the positions of the governments within indicated intervals, such that the relative distances between the governments in the MacBeth scale adequately represent the party's relative distances of attractiveness between these governments with respect to the given criterion c.

The relative weights $\alpha_i(c)$ can also be determined by MacBeth in a similar way.

$$U_i(g) = \alpha_i(c_0)u_i(c_0, g) + \ldots + \alpha_i(c_m)u_i(c_m, g)$$

3. Applying the RelVieW software
to determine the stable governments

Agnieszka Rusinowska developed an example based on the structure of Polish government after the 2001 elections. In this example there are 7 parties, 8 different policies and 120 governments, 17 of which are feasible (to be computed by RelVieW); only 4 parties are involved in them; together forming the set DM:
SLD, Alliance Democratic Left & Labor Union
PO, Civil Platform
PiS, Self-Defense, Law and Justice, and
PSL, Polish Peasant Party.

Input for RelVieW: *government-membership relation* M as Boolean 4 x 17 matrix:

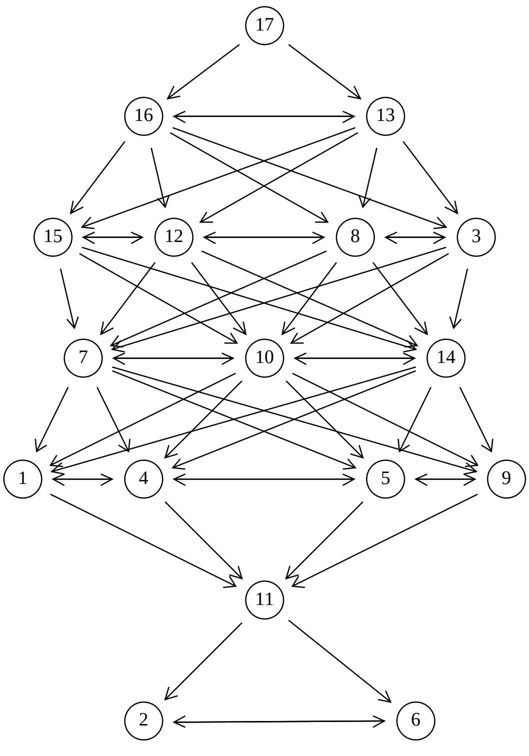
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
SLD	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
PO	■		■		■			■		■			■		■		■
PiS		■		■		■			■		■			■			
PSL							■					■				■	

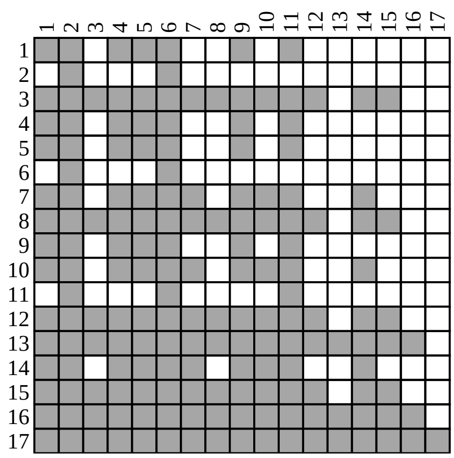
$M : DM \leftrightarrow G^*$ where $M_{i,g} \leftrightarrow i \in S$ if $g = (S, p)$.

Input for RelVieW: for each $i \in DM$ the *utility relation* $R^i : G^* \leftrightarrow G^*$ or rather the *Comparison relation* $C : DM \leftrightarrow G^* \times G^*$ defined by

$$R^i_{h,g} \leftrightarrow U_i(h) \geq U_i(g) \text{ and } C_{i,<h,g>} := R^i_{h,g}$$

In our Polish example, for instance, given the utilities of the 17 feasible governments for SLD, computed by MacBeth, R^{SLD} can be represented in RelVieW graphically as follows, expressing that $U_{SLD}(g_{17}) \geq U_{SLD}(g_{16})$, etc.:





Graph, resp. matrix, for R^{SLD}

$dominance(M, C)_{h,g}$:= for all i, $M_{i,h} \to C_{i,<h,g>}$ and for some i, $M_{i,h}$ & $\neg C_{i,<g,h>}$.

In relation-algebraic terms: $dominance(M, C)$

$$= \overline{(\pi; M^{\mathsf{T}} \cap \overline{C}^{\mathsf{T}}); \mathsf{L}} \cap (\pi; M^{\mathsf{T}} \cap E; \overline{C}^{\mathsf{T}}); \mathsf{L} \qquad (3)$$

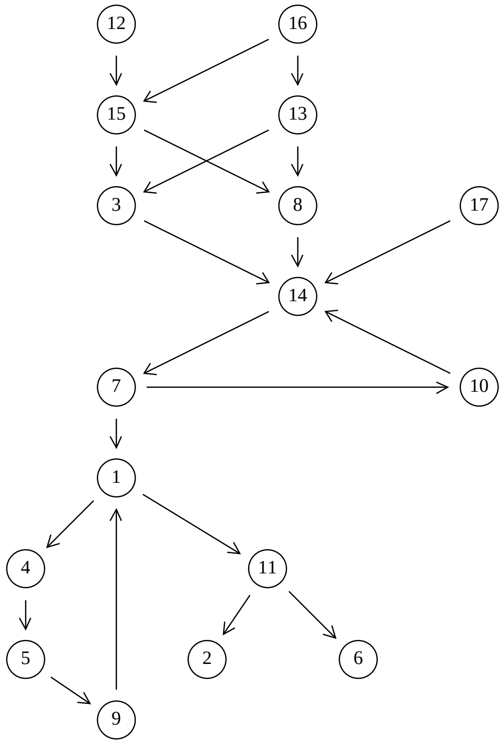
where π and ρ are projection relations, L is the universal relation, E is the exchange relation for pairs, M^{T} denotes the Transposition of M and ; denotes composition.

$stable(M, C)_g$:= there is no h such that $dominance(M, C)_{h,g}$.

In relation-algebraic terms:

$$stable(M, C) = \overline{\rho^{\mathsf{T}}; dominance(M, C)}. \qquad (4)$$

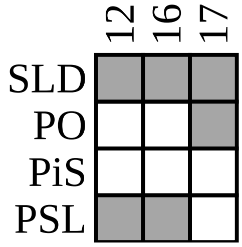
In the Polish example, given input M and C, RelVieW computes the $dominance(M, C)$ and the $stable(M, C)$ relation and can give a graphical representation of them:



Graph for the dominance relation $\succ$

According to this directed graph 3 out of 17 feasible governments are stable, viz. the sources /governments g_{12}, g_{16}, and g_{17}.

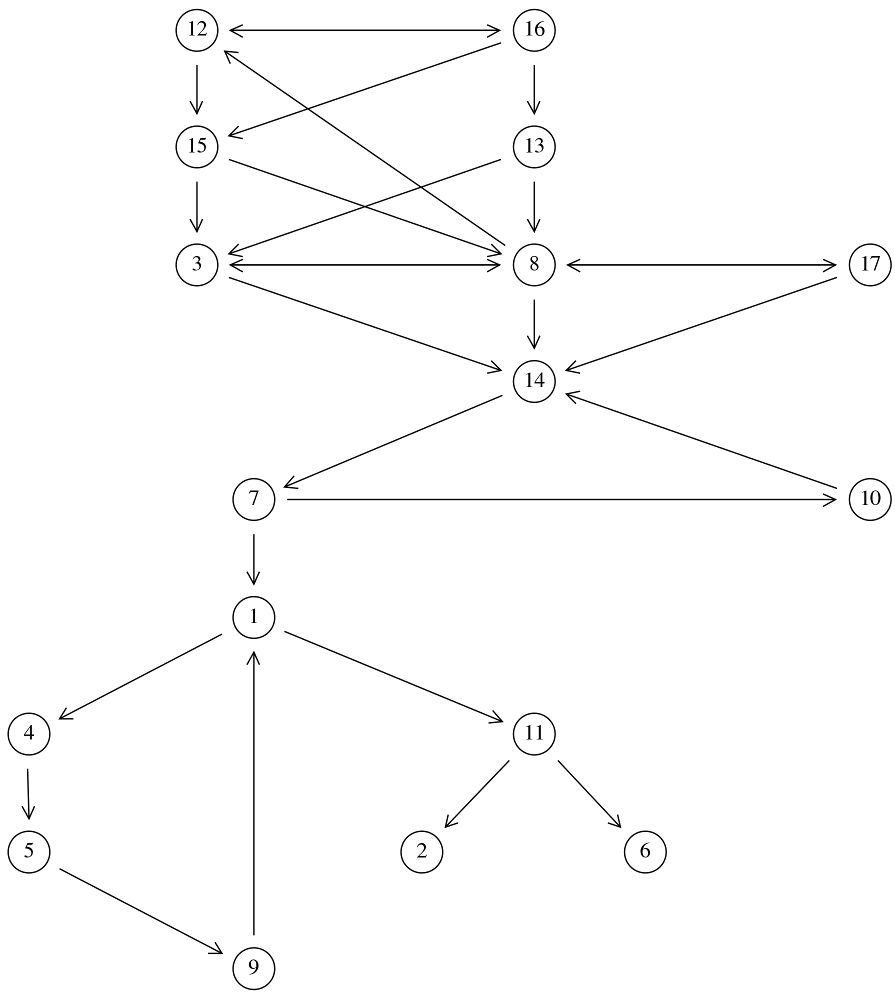
In the Polish example, RelView can represent the stable governments by the matrix:



Applying Graph Theory and RelVieW
in case there is no stable government

Which government should be chosen when the dominance graph has no source?

Example:



Use the following concepts from graph theory:

strongly connected components (SCCs) - maximal sets of vertices such that each pair of vertices is mutually reachable.

The SCCs of our input graph are $\{1, 4, 5, 9\}$, $\{2\}$, $\{6\}$, $\{11\}$, $\{7, 10, 14\}$, and $C_3 = \{3, 8, 12, 13, 15, 16, 17\}$.

Initial SCCs - SCCs without arcs leading from outside into them. The governments of an initial SCC can be seen as a cluster which is not dominated from outside.
In our example, the only initial SCC is C_3.

minimal feedback vertex set (FVS) - a minimal set of vertices that contains at least one vertex from every cycle of the graph.

C_3 contains the cycles: $\{12, 16\}$, $\{3, 8\}$, $\{8, 12, 15\}$, $\{3, 8, 12, 15\}$, $\{8, 12, 16, 13\}$, $\{8, 12, 16, 15\}$, $\{3, 8, 12, 16, 15\}$, $\{8, 17\}$
The two minimal FVSs of C_3 : $\{8, 12\}$, $\{8, 16\}$.

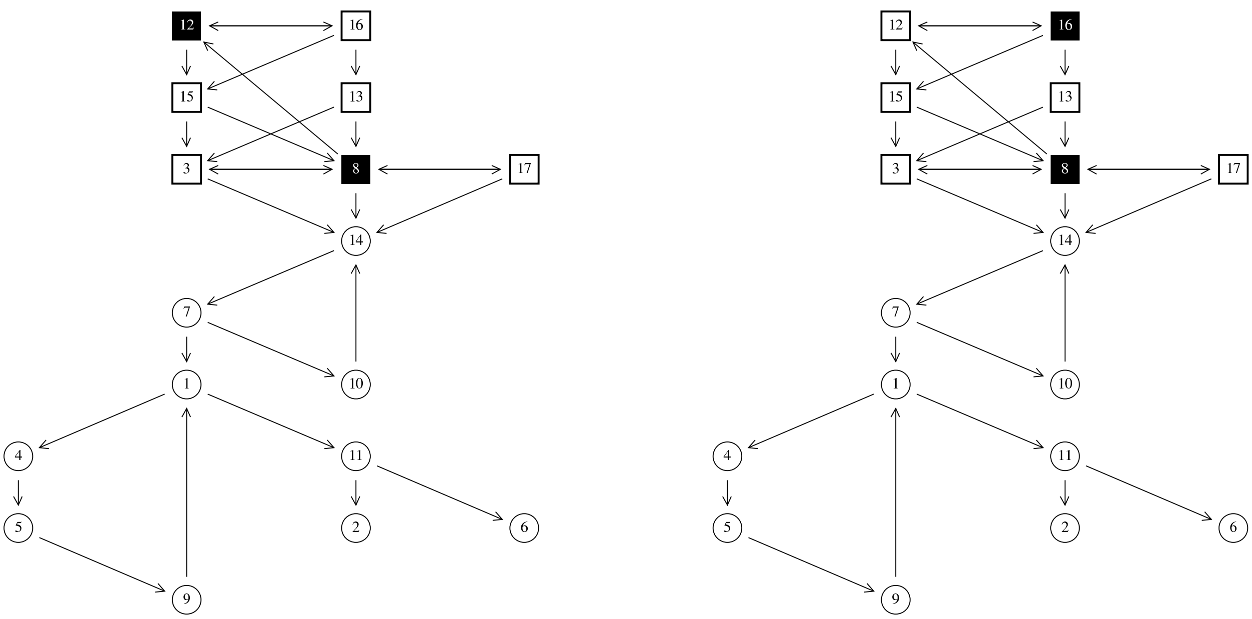
Procedure

Step 1. Compute the set $\mathfrak{I}$ of all initial SCCs of the dominance graph.
In our example, $\mathfrak{I} = \{C_3\}$, where

$$C_3 = \{3, 8, 12, 13, 15, 16, 17\}.$$

Step 2a. For each initial SCC C from $\mathfrak{I}$ compute the set $\mathfrak{F}$ of all minimal FVSs of the subgraph generated by the vertices of C.
In our example, $\mathfrak{F} = \{\{8, 12\}, \{8, 16\}\}$.

The original graph marked with the minimal FVSs:



Step 2b. Select from all sets of $\mathfrak{F}$ with a maximal number of ingoing arcs one with a minimal number of outgoing arcs. We denote this one by F.

In our example, the minimal FVS $\{8, 12\}$ has 5 ingoing arcs, while $\{8, 16\}$ has only 4 ingoing arcs. So, {8, 12} is most frequently dominated and $F = \{8, 12\}$.

Step 2c. Break all cycles of C by removing the vertices of F from the dominance graph. This corresponds to a removal of those candidates which are 'least attractive' for two reasons: because they are most frequently dominated and they dominate other governments least frequently.

Hence, in our example we remove the vertices 8 and 12 from the graph, which leads to 16 and 17 as new sources, i.e., as governments that can be considered as rather stable.

Step 2d. Select an un-dominated government from the remaining graph. If there is more than one candidate, use bargaining or social choice rules in order to choose one.

In our example: SLD and PSL are involved in g_{16} and SLD and PO in g_{17}.
$U_{SLD}(g_{17}) > U_{SLD}(g_{16})$, $U_{PO}(g_{16}) > U_{PO}(g_{17})$, $U_{PSL}(g_{16}) > U_{PSL}(g_{17})$. So, applying Plurality, Majority or Borda rule yields g_{16}.

Step 3. If there is more than one set in $\mathfrak{I}$, select the final stable government from the results of the second step by applying bargaining or social choice rules again.

All computations can be executed by the RelVieW tool, up till about 100 nodes.

URLs: MacBeth: www.M-MACBETH.com
RelVieW: http://www.informatik.uni-kiel.de
/~progsys/relview/

COMSOC.tex