

Tutorial on Fairness and Uncertainty

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Decision Theory (and especially decision theory under risk and uncertainty) and Social Choice are intimately connected, both at a conceptual and at a formal level. This tutorial aims at showing various connections between these two fields. General references on Social Choice are Roemer (1996), Kolm (1995) and, for a shorter survey, Sen (1986). Concerning Decision Theory, Karni and Schmeidler (1991) provide a nice introduction.

1 General Remarks on Decision Theory and Ethics

Decision Theory is a *normative* theory, that tries to figure out what a *rational* behavior (i.e., a goal-directed and consistent behavior) should be. Social Choice Theory is also a *normative* theory, that tries to figure out what a *moral* behavior should be. But, as stated by Harsanyi (1992):

Indeed, most philosophers also regard *moral behavior* as a special form of rational behavior. If we accept this view (as I think we should) then the theory of morality, i.e, moral philosophy or ethics, becomes another normative discipline dealing with rational behavior.

Of course, such a view is not undisputed (see, e.g., Sen (1995) for a discussion). But it is also certainly not absurd, and we will try to see its consequences. A first, obvious, consequence, is that Social Choice, as a theory of rational choice, is part of Decision Theory. The second part of the talk will be devoted to that connection. More precisely, we will show how most mathematical objects of individual decision theory under risk and uncertainty can be interpreted in a social choice framework. We will show how usual tools of decision theory can then be used to solve social choice problems.

*This is an extended and preliminary abstract. It should be considered as a very first draft. Most likely, the talk will not cover all the material presented here.

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But there is another deep link between decision theory and ethics. Indeed, consider the classical problem of allocating an indivisible item between two persons. The conventional wisdom is to let a fair coin decide who will get the good. Uncertainty thus plays a fundamental role in our intuitive perception of fairness. The third part of the talk will be devoted to a formal analysis fairness as uncertainty.

Finally, most of the social choice literature in economics consider that what is actually relevant in collective decisions is individuals' preferences. Social choice should be considered as an attempt of conciliate individuals' preferences into a collective one. The last part of the talk will be devoted to the study of this problem, when alternatives under choice are risky or uncertain.

We present a road map of the talk, with bibliographical references.

2 Fairness and Uncertainty: Objects

2.1 Lotteries and Income Distributions

Lotteries and income distributions can be shown to be the same mathematical objects. A large literature, devoted to inequality measurement, is based on this observation: choices between income distributions are then seen as choices between lotteries. In particular, *risk-aversion* can be shown to be equivalent to *inequality-aversion*. More precisely, a mean-preserving spread on lotteries corresponds to an income transfer from a poorer to a richer in an income distribution. Important contributions in this field are: Atkinson (1970), Sen (1973), Kolm (1976a), Kolm (1976b), Weymark (1981), Ebert (1988).

2.2 Anscombe-Aumann acts and uncertain income distributions

Anscombe and Aumann (1963) introduced a representation of uncertainty that is extensively used in individual decision theory. A set of states of the nature S and a set of consequence X are given. Let $\Delta(X)$ be the set of simple probability distributions on X . An *act* is a function from S to $\Delta(X)$. The interpretation is the following: you do not know what are the probabilities of each states (uncertainty), but you know that, if a given state s occurs, you'll get a well-defined lottery $f(s)$.

Extending the parallel we made between lotteries and income distributions to this framework, we will show that we can interpret an Anscombe-Aumann act as an uncertain income distribution. We will use this framework to discuss the very important notions of *ex ante* and *ex post* fairness. Relevant references are Ben Porath, Gilboa, and Schmeidler (1997) and Gajdos and Maurin (2004).

3 Fairness as Uncertainty

3.1 Overview: The Rawls-Harsanyi debate

In this part of the talk, we will discuss the role of uncertainty as a tool to elaborate fair decisions. Rawls (1971) and Harsanyi (1953), (1977), promoted the idea that the principle of justice are those that a rational decision maker would choose under the appropriate conditions of impartiality. Both argued that the “Veil of Ignorance” was precisely the right tool to model this impartiality requirement. Under such a veil, the decision maker is deprived of any information related to herself, her personal situation and her personal features. In particular, she makes her choices without knowing *who* she is, and *what place she occupies in the society*. In other words, under the veil of ignorance, the decision maker is *uncertain* about her own actual identity and position in the society. Both Rawls and Harsanyi argue that a rational decision under these information restrictions will be fair. However, if they agree about the assumptions, they sharply disagree about the conclusion that can be drawn from it, since Harsanyi claims that the rational decision criterion under such informational constraint should be of the expected utility form (with uniform distribution), whereas Rawls argues in favor of a maxmin criterion. An extensive discussion on this subject might be found in Sen (1986) and Weymark (1991).

3.2 The Ignorant Observer

We will present a model, inspired by recent developments in decision theory under uncertainty (Gajdos, Tallon, and Vergnaud (2004)) that allows to encompass Harsanyi’s and Rawls’ views in a common framework. This is an ongoing work, joint with Ferial Kandil. It rests on the idea that uncertainty should be represented as a *set* of probability distributions *and* an Anscombe-Aumann act (the role of Anscombe-Aumann acts in the analysis of impartial decisions has been pointed out by Karni and Weymark (1998)). It will then be argued that epistemic assumptions are not enough to characterize a precise collective decision rule, and should be supplemented with ethical assumptions. Hence, after all, social choice is maybe not just applied decision theory.

4 Fairness under Uncertainty

In this section, we will consider the idea that social choice consists in conciliating individuals’ preferences, so to promote common good. We will consider two way of doing this: first, by simply considering the possibility of aggregating individuals’ preferences; second, by considering the possibility of obtaining envy-free allocations. We will restrict our attention to situations in which individuals’ preferences are defined over lotteries or Anscombe-Aumann acts.

4.1 Aggregation under Risk

Arrow's celebrated Impossibility Theorem (Arrow (1951)) establishes that, under mild conditions, there is no way to aggregate individuals' preferences in a non-dictatorial way. However, this result rests on the so called *Universal Domain Assumption*, which imposes that no restriction is made on individuals' preferences. Once one relaxes this assumption, the situation becomes very different. Indeed, Harsanyi (1955) show the following striking result: assume that the society, as well as all individuals, are expected utility maximizers. Then, the social decision rule respects individuals' preferences (Pareto condition) if, and only if, it is a convex combination of individual expected utilities.

4.2 Aggregation under Uncertainty

It is a natural question to wonder if Harsanyi's Aggregation Theorem is still valid under uncertainty. We will show that such is not the case. More specifically, assume we impose some common rationality postulates on both collective and individual preferences (these conditions are weak enough to encompass most of known decision models under uncertainty). Then, as soon as individuals exhibit some kind of uncertainty aversion, the collective decision rule will satisfy the Pareto condition if, and only if, it is dictatorial (Gajdos, Tallon, and Vergnaud (2005)). This result suggests that the notion of "collective rationality" is not as innocuous as it might seem.

4.3 Notes on Envy and Uncertainty

Finally, we will turn to the question of the existence of envy-free allocations when individuals are expected utility maximizers. We will define two concepts of envy-freeness (*ex ante* and *ex post*), and will show that there is in general no allocation that is both *ex ante* and *ex post* envy-free and efficient. However, a possibility result can be achieved when one considers economies without aggregated risk (Gajdos and Tallon (2002)).

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