Optimal Outcomes of Distributed Negotiation in Utilitarian and Egalitarian Settings

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Talk Overview

1. Distributed Resource Allocation
   - MARA...the setting
   - Our Framework: Main definitions

2. Experiments

3. Egalitarian Social Welfare
   - Theoretical and experimental results
   - Condition under which egalitarian $SW=0$
   - Limit
Outline

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Resource Allocation Framework
- Finite set of agents $A$ and finite set of discrete resources $R$

Definition (Allocation)
An allocation of resources for the system $(A, R)$ is a function $A$ from agents in $A$ to subsets of $R$ such that $A(i) \cap A(j) = \{\}$ for $i \neq j$ and $\bigcup_{i \in A} A(i) = R$.
Restriction on deal type: Bilateral deals
⇒ One-resource-at-a-time (from an agent to another)

Each agent $i \in \mathcal{A}$ has a utility function $u_i$ to express his personal welfare

Example (k-additive functions: multinomial of degree $k$)

$u_1(R) = r_1 + 3r_2 + 7r_3 \Rightarrow 1$-additive
$u_2(R) = 2r_1 + 3r_2 + 7r_1r_3 \Rightarrow k$-additive: $2$-additive
$u_2(r_1, r_3) = 9$
How to measure social welfare?

- well-being of the society
- 2 classical measures of Social Welfare

**Definition (Utilitarian Social Welfare)**

\[ sw_u(A) = \sum_{i \in A} u_i(A) \]

**Definition (Egalitarian Social Welfare)**

\[ sw_e(A) = \min\{u_i(A)\} \]
Agents and rationality

- individual rationality

**Lemma (Rational deals)**

A deal \( \delta = (A, A') \) is individually rational iff \( sw_u(A) < sw_u(A') \)

- payment function

**Definition (Payment function)**

\[ \sum p(i) = 0 \]

- We assume that money is unlimited
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We have some theoretical results
- AAMAS 03: In rational negotiation, with one-deals we know that we rise to the utilitarian optimum.
- What about the others optimum?

Why experiments?
- for better understanding theoretical results
- to induce new results
  ⇒ We know that we reach some results like optimal Utilitarian Social Welfare but we want to know in other cases

Test all the kinds of Social Welfare
- egalitarian
- utilitarian
Experimental protocol

- During experimentation, variation of some parameters
  - number of resources
  - number of agents
  - Complexity of the utility function

- 4 steps
  1. System creation
  2. Exhaustive search of the optimal allocation
  3. Negotiation until no more deal is possible
  4. Change parameters and Go to 1
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Egalitarian Social Welfare: theoretical results

Theorem (Bouveret & al, AAMAS 05)

Even with additive utility, to find the egalitarian optimum is a NP-hard problem.

- \#Resource = 10
- \#Term = 1..10
- \#Agent = 2
- k=1
Egalitarian Social Welfare: experimental results

- When the number of agents rise, the poorest is poor and poor.
- When will Egalitarian Social Welfare = 0?

- \#Resource = 5
- \#Term = 5
- \#Agent = 1..7
- k=1
Lemma (shortage of resource)

If the number of agents exceeds the number of resources, then one of the agents will be necessarily deprived of resource, in which case $sw_e = 0$.

Lemma (generalization to $k$-additive function)

If an agent requires for at least $k$ resources to have a non-null utility, the egalitarian SW will be null if $k_{\text{min}} > \frac{\#\text{Resource}}{\#\text{Agent}}$.

Example ($k=3$)

3 agents, 2 resources

- $u_1 = r_1$
- $u_2 = 3r_1$
- $u_3 = 4r_2$

The agents couldn’t be satisfied in the same time, so one agent had a utility $= 0$. 
Condition under which egalitarian $SW=0$ (2)

Example

\[
\begin{align*}
    u_1 &= r_1 r_2 \\
    u_2 &= r_2 r_3 + r_4 \\
    u_3 &= r_3 r_1 + r_4
\end{align*}
\]

When an agent have a utility $> 0$, the other has to be 0.

We can see that there exist a blocking situation which is different than shortage of resource.
Limit for Egalitarian Social Welfare

Example ($\alpha_{max}$)

\[ u_1 = 2r_1 + 4r_2 \text{ and } u_2 = r_1 \]
\[ \Rightarrow \text{ then } \alpha_{max} = 4 \]

Lemma (limit with k=1)

\[ sw_e \leq \frac{\#Resource}{\#Agent} \times \alpha_{max} \]
\[ sw_e \leq \#terms \times \alpha_{max} \]

with $\alpha_{max}$ maximal coefficient of all the utility functions

Lemma (limit for all k)

\[ sw_e \leq \frac{\#Resource \cdot \#Term}{k \cdot \#Agent} \times \alpha_{max} \]
condition under which egalitarian $SW=0$ and limit

comparaison des $SW$ des allocations finales et optimaux pour les $SW$ egalitaire en fonction du nombre d'agent.

- allocation optimale egalitaire
- allocation finale egalitaire
- allocation initiale egalitaire
- limit

$SW_e=0$
Methodological question

- To what extant does it make sense to assess the egalitarian welfare when payment are allowed?
  
  - When we are interested only by the quality of the allocation and when payments are only virtual.
  
  - Run the experiments with a payment function

Example (Uniform payment function)

\[ p(i) = u_i(A') - u_i(A) - \frac{sw_u(A') - sw_u(A)}{|\delta A|} \quad \text{for } i \in \delta A \]
Conclusion and Future work

- **Conclusion**
  - We found a limit and a condition where Egalitarian Social Welfare $= 0$

- **Future work**:
  - More tests with various parameters
  - Studying Utilitarian social welfare
  - Studying swap deals and other deal types