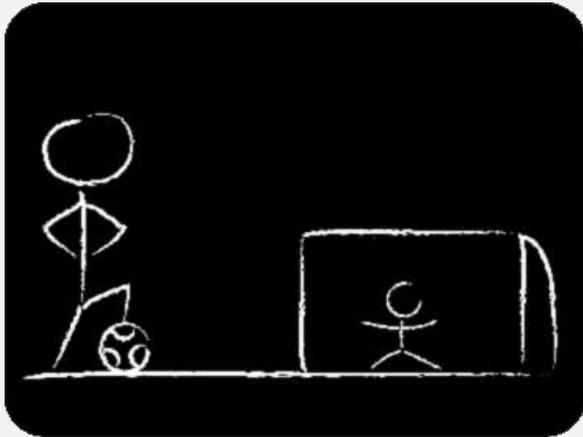


Computational Aspects of Manipulation and Control in Judgment Aggregation

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ESSLLI Workshop on Logical Models of Group Decision Making
August 2013

Judgment Aggregation



	Penalty Area	Foul	Penalty
	Yes	Yes	Yes
	Yes	No	No
	No	Yes	No
Majority	Yes	Yes	No



Doctrinal Paradox / Discursive Dilemma

- Formal Framework
- Manipulation
 - Types of preferences
 - Strategyproofness
 - Complexity Results
- Control
 - Bundling Judges
 - Complexity Results

Formal Framework

Judges



	Penalty Area	Foul	Penalty
Referee 1	Yes	Yes	Yes
Referee 2	Yes	No	No
Referee 3	No	Yes	No
Quota $\frac{1}{2}$	Yes	Yes	Yes

Individual Judgment Sets

Yes / No

Quota

fraction for each premise

Collective Judgment Set

Yes if quota is reached

Requirements:

- Agenda is closed under propositional variables
 - Premises consists of all literals
- ⇒ Complete and consistent outcome

Variants:

- Uniform quota
- Constant quota

We focus on:

- PBP: Uniform premise-based quota rule for quota $\frac{1}{2}$

Forms of Interference

Manipulation:

Provide untruthful information to obtain a better result.

Bribery:

Briber judges to obtain a better result.

Control:

Change the structure to obtain a better result.

Widely studied in voting from a computational point of view!

Incentive:

Provide untruthful information to obtain a better result.

- Information = individual judgment set
- Result = collective outcome
- Better = ?

Different assumptions on the preferences:

- Unrestricted
- Top-respecting
- Closeness-respecting
- Hamming-distance induced

Preferences over collective JS

Preferences with respect to JS

1	0	0	1	1
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- Unrestricted (U): every preference is possible

- Top-respecting (TR):

1	0	0	1	1
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?	?	?	?	?
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- Closeness-respecting (CR):

1	?	?	?	1
---	---	---	---	---

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1	1	1	0	1
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- Hamming-distance induced (HD):

$$\begin{array}{|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 1 \\ \hline \end{array} > \begin{array}{|c|c|c|c|c|} \hline 1 & 1 & 1 & 0 & 1 \\ \hline \end{array}$$

The only complete relation is HD (by allowing equalities)

Beispiel

	a	b	c	d		$a \vee b$	$b \vee c$	$a \vee c$	$b \vee d$
Judge 1	1	1	0	0		1	1	1	1
Judge 2	0	0	0	0		0	0	0	0
Judge 3	1	0 1	1	1		1	1	1	1
PBP	1	0 1	0	0	\Rightarrow	1	0 1	1	0 1

Manipulation b

- Unrestricted: ?
- Top-respecting: ? but Conclusions preferred
- Closeness-respecting: ? but Conclusions preferred
- Hamming-distance induced: preferred

Fix some induced preference \succ :

A judge **necessarily** prefers X to Y if $X \succ Y$ in *every* complete extension of \succ .

A judge **possibly** prefers X to Y if $X \succ Y$ in *some* complete extension of \succ .

A judgment aggregation procedure is **necessarily/possibly strategyproof** if a judge necessarily/possibly prefers the actual outcome to all outcomes resulting from untruthful individual judgment sets of him.

Manipulation

A	F	$A \wedge F$
Yes	Yes	Yes
Yes	No	No
No	Yes	No
Yes	Yes	Yes

Manipulative judge

Question: Is it possible to obtain a „better outcome“ by reporting an insincere judgment set?

A	F	$A \wedge F$
Yes	Yes	Yes
Yes	No	No
No	No	No
Yes	No	No

HD, TR, CR-preferences
regarding $A \wedge F$, Exact

Results for Manipulation

Preferences	Necessary Manipulation	Possible Manipulation
Unrestricted	?	in P
Top-respecting	NP-complete	in P
Closeness-respecting	strategyproof	?
Hamming-distance	NP-complete	
Exact	strategyproof	

Complete desired
judgment set

Also holds for general quotas

Incentive:

Change the structure to obtain a better result.

Different types of control

- Adding Judges
- Deleting Judges
- Replacing Judges

- Bundling Judges

Focus on exact and Hamming-distance variant

Control by Bundling Judges

A	F	$A \wedge F$
Yes	Yes	Yes
Yes	No	No
No	Yes	No
Yes	Yes	Yes

No

Partition into 2
groups (A and F)

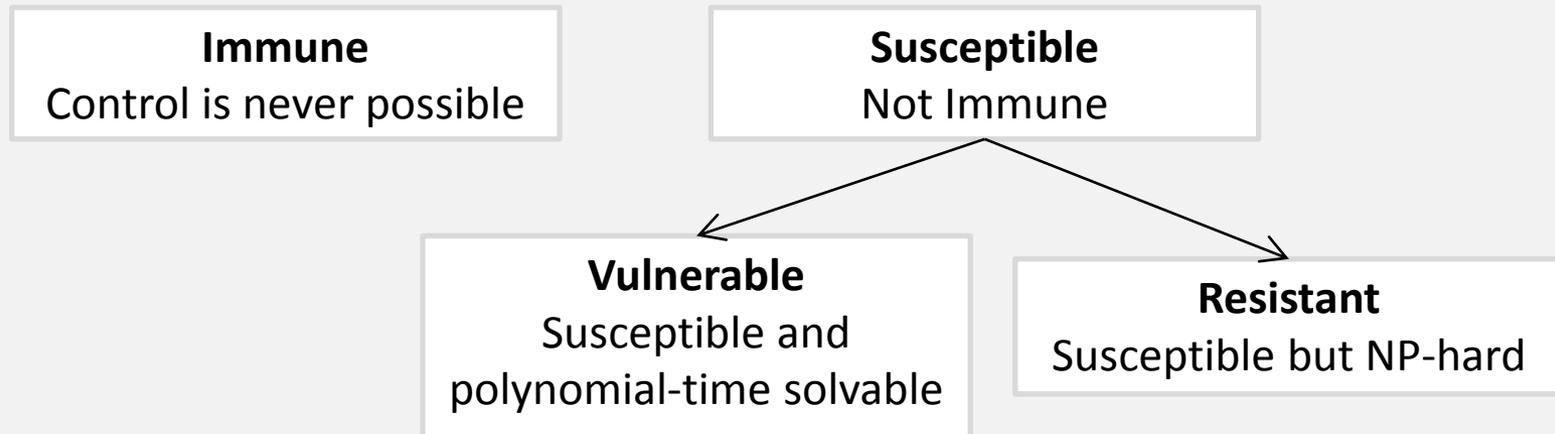
A	F	$A \wedge F$
Yes	Yes	Yes
Yes	No	No
No	Yes	No
No	No	No

- Desired judgment set
- Partition of the premises

Question: Is it possible to obtain a „better outcome“ by partitioning the judges?

Exact Variant: Is it possible to reach the desired judgment set by partitioning the judges?

Control is usually an undesired behavior



⇒ Computational hardness can be seen as a barrier against control

Results for Control

	Constant Number	PBP	Uniform Quota
Adding Judges (HD)	resistant	resistant	
Adding Judges (Exact)	resistant	resistant	
Deleting Judges (HD)	resistant	resistant	
Deleting Judges (Exact)	resistant	resistant	
Replacing Judges (HD)	resistant	resistant	resistant
Replacing Judges (Exact)	resistant	resistant	resistant
Bundling Judges (HD)	Def?	resistant	
Bundling Judges (Exact)	Def?	resistant	

Reduction from
Dominating Set

Reduction from
Deleting Judges (Exact)

Reduction from
Exact Cover by 3-Sets

Reduction
Exact to HD variant

Agenda contains only
premises

Concluding Remarks

- Different Aggregation Procedures
- New Control Problems
- Typical-case analysis
- Different types of induced preferences for Bribery and Control

Thank you for your attention!