Logical Models of Group Decision Making

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Motivation for this Workshop

Recent trend in research communities such as philosophical logic, AI, and theoretical computer science to engage in the formal study of problems originating in economics. Proliferation of keywords such as:

- logic and rational interaction
- computational social choice

But interaction between those taking logic as their point of departure and those taking economic theory as theirs is still limited.

With this workshop, we wanted to address this lack of interaction, focussing on a specific domain:

- the use of *logic* (rather than formal methods more generally)
- to model problems in *group decision making* (rather than problems in economics more generally)

Main Topics at this Workshop

We accepted 10 out of 14 submissions for presentation. Main topics:

- *judgment aggregation, including computational aspects*
  Nehring & Pivato, Baumeister et al., Jamroga & Slavkovik
- *model theory and proof theory applied to social choice*
  Eckert & Herzberg, Poliakov, Maffezioli
- *logics for modelling individuals (possibly within a group)*
  Goranko & Bulling, Rendsvig

Speakers will include mathematical and philosophical logicians, economic theorists, and computer scientists.

Outline of this Talk

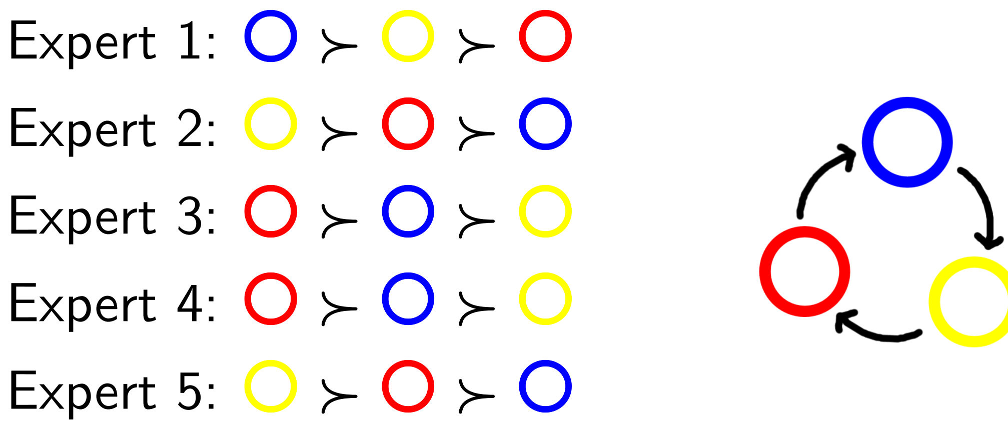
I will begin with a brief introduction to *social choice theory*, the study of methods for group decision making.

I will then discuss three applications of *logic* to social choice theory:

- the *axiomatic method*, as understood in in economic theory
- *modelling* social choice problems in symbolic logic
- *judgment aggregation* ("logical aggregation")

Classical Example: The Condorcet Paradox

Social choice theory asks: how should we aggregate the preferences of the members of a group to obtain a "social preference"?



Marie Jean Antoine Nicolas de Caritat (1743–1794), better known as the **Marquis de Condorcet**: Highly influential Mathematician, Philosopher, Political Scientist, Political Activist. Observed that the *majority rule* may produce inconsistent outcomes ("Condorcet Paradox").

Classical Result: Arrow's Impossibility Theorem

In 1951, K.J. Arrow published his famous *Impossibility Theorem:*

Any preference aggregation mechanism for *three* or more alternatives that satisfies the axioms of *Pareto* and *IIA* must be *dictatorial*.

- Pareto: if everyone says $X \succ Y$, then so should society.
- Independence of Irrelevant Alternatives (IIA): if society says $X \succ Y$ and someone changes her ranking of Z, then society should still say $X \succ Y$.

Kenneth J. Arrow (born 1921): American Economist; Professor Emeritus of Economics at Stanford; Nobel Prize in Economics 1972 (youngest recipient ever). His 1951 PhD thesis started modern Social Choice Theory. Google Scholar lists 12906 citations of the thesis.

The Axiomatic Method

Many important results in social choice theory are *axiomatic*.
They formalise desirable properties as "*axioms*" and then establish:

- *Impossibility Theorems*, showing that there exists *no* aggregation mechanism satisfying a given set of axioms
- *Characterisation Theorems*, showing that a particular (class of) mechanism(s) is the only one satisfying a given set of axioms

Seminal results include:

- Arrow (1951): impossible to do nondictatorial preference aggregation that is Pareto and independent (for $\geqslant 3$ alternatives)
- May (1952): the simple majority rule for two alternatives is fully charaterised by anonymity, neutrality, and positive responsiveness
- Gibbard-Sattherthwaite (1973/75): impossible to do nondictatorial resolute voting with $\geqslant 3$ possible winners that is strategy-proof

Logics for Social Choice Theory

The *axiomatic method* in SCT does borrow some terms from logic ("axiom", "inconsistent") and it will appeal to the logician, but it does not really use *symbolic logic* (no formal language, no inference rules).

We may want to develop *logics for social choice*. Reasons:

- Formalisation deepens *understanding*.
- Just as logic has been used to *verify* computer systems, we may want to try the same for social choice mechanisms.
- The *expressivity* needed to specify a property tells us something interesting about the underlying concept (e.g., do we need second-order quantification to speak about IIA?).

Modelling the Arrovian Framework

We can model Arrovian preference aggregation in *first-order logic*.

Important trick: introduce *situations* to refer to *profiles*. Examples:

- Transitivity: $\forall z, x_1, x_2, x_3, u.[I(z) \wedge A(x_1) \wedge A(x_2) \wedge A(x_3) \wedge S(u) \rightarrow (p(z, x_1, x_2, u) \wedge p(z, x_2, x_3, u) \rightarrow p(z, x_1, x_3, u))]$
- IIA: $\forall u_1, u_2, x, y.[S(u_1) \wedge S(u_2) \wedge A(x) \wedge A(y) \rightarrow [\forall z.(I(z) \rightarrow (p(z, x, y, u_1) \leftrightarrow p(z, x, y, u_2))) \rightarrow (w(x, y, u_1) \leftrightarrow w(x, y, u_2))]]$

Arrow's Theorem now reduces to this:

Theorem: $\mathrm{T}_{\mathrm{PA}} \cup \{\mathrm{PAR}, \mathrm{IIA}, \mathrm{NDIC}\}$ *has no finite model.*

Related work: (new) *modal logic* (Ågotnes et al., JAAMAS 2011); *propositional logic* (Tang & Lin, AIJ 2009); *HOL* (Nipkow, JAR 2009). For the latter two the focus is on automated reasoning.

U. Grandi and U. Endriss. First-Order Logic Formalisation of Impossibility Theorems in Preference Aggregation. *J. Phil. Log.*, 42(4):595-618, 2013.

Research Challenges

What is the "right" logic to model social choice?

- don't fix the *set of individuals* (and alternatives) in the language
- model the *universal domain* assumption in an elegant manner
- support *automated reasoning*

How far can we push automation of reasoning about social choice?

- *full automation* *vs*. interactive theorem proving / ground instances
- *verification* of results in SCT and *discovery** of new theorems
- support of *practical reasoning* about concrete mechanisms

*For a very simple area of SCT ("ranking sets of objects") we managed to achieve fully automated discovery of theorems.

C. Geist and U. Endriss. Automated Search for Impossibility Theorems in Social Choice Theory: Ranking Sets of Objects. *JAIR*, 40:143–174, 2011.

Judgment Aggregation

Three agents hold different views on the truth of the propositions p, $p \to q$, and q (e.g., p might stand for "the temperature is above 20°C" and q for "we should open the window").

Think of p and $p \to q$ as the *premises*, and of q as the *conclusion*.

	p	$p \to q$	q
Agent 1:	Yes	Yes	Yes
Agent 2:	Yes	No	No
Agent 3:	No	Yes	No

What should be the *collective* decision?

Aggregation rules to try: premise-based, conclusion-based, majority

Agenda Characterisation

The *agenda* is the set of formulas (closed under complementation) which our agents are asked to judge ...

One important research direction in JA has been to characterise those agendas for which the result of aggregation will always be consistent.

Example for a result (Nehring and Puppe, 2007):

> The *majority rule* will return a *consistent* judgment set for every possible profile on agenda Φ *if and only if* it is the case that every *minimally inconsistent subset* of Φ has *size* $\leqslant$ *2*.

Btw: deciding this property is *highly intractable* ($\mathrm{coNP}^{\mathrm{NP}}$-complete).

K. Nehring and C. Puppe. The Structure of Strategy-proof Social Choice. Part I: General Characterization and Possibility Results on Median Space. *Journal of Economic Theory*, 135(1):269–305, 2007.

U. Endriss, U. Grandi, and D. Porello. Complexity of Judgment Aggregation. *Journal of Artificial Intelligence Research*, 45:481–514, 2012.

Research Challenges

To date, most work on judgment aggregation has used the axiomatic method to derive possibility and impossibility results.

But work on more basic questions is still fairly rare:

- develop more *concrete rules* and understand their features
- design *algorithms*
- improve modelling of *strategic behaviour*
- explore *logic* component of the model (e.g., use proof theory)
- develop *applications* (ideas: multiagent systems, ontology merging, collective annotation in computational linguistics, ...)

Last Slide

I have talked about three applications of *logic* to *social choice theory:*

- the axiomatic method
- logical modelling
- judgment aggregation

For a more systematic overview, feel free to attend *my course* (at 11:00):

- This morning: preference aggregation, Arrow's Thm
- Tuesday: voting, May's Thm, Gibbard-Satterthwaite Thm
- Wednesday: logical modelling in propositional, modal, first-order logic
- Thursday/Friday: judgment aggregation

Rationale for this workshop: logicians talking about "social interaction" etc. should have a closer look at SCT and see whether they can really model things at the mechanism level + people working in SCT should explore whether more sophisticated tools from logic might help them.

U. Endriss. Logic and Social Choice Theory. In A. Gupta and J. van Benthem (eds.), *Logic and Philosophy Today*, College Publications, 2011.