Expressive Voting: Modelling a Voter’s Decision to Vote
Workshop on Logical Models of Group Decision Making

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Content of the talk

- Relationship between Voting Theory and Rational Choice Theory
- Two explanatory schemes for voting: expressive vs. instrumental.
- Expressive voting-based analysis of voting systems
- Discuss a current approach by Gilboa et al. and present an alternative
Voting and Rational Choice

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Slogan: **Utility is the Utility of the outcome**
Voting and Rational Choice

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- Voting as an instrument to influence outcome Instrumental Account of Voting
- Strategic Considerations prominently studied in voting theory: Gibbard Satterwaithe
Prominent criticism:

(Downs 1957): Extend the image of the rational voter by taking into account the cost $L$ for going to the election booth. Leave home if

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Why do people vote?
Expressive Voting

Prominent disanalogy to Economic Reasoning:

- In economic interactions: expressing the preference ensures the outcome (buying a car...)
- Revealed preference deals with outcomes only. (Mostly...)
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- Prominently accepted answer: The fact of voting itself is an act that produces utility.

Thus \[ L \leq h \cdot R + E \]

Where \( E \) is the utility of the expressive act

See Brennan/Lomasky (1993) for a deeper discussion

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Study both kinds of motivations separately to understand voting behaviour.
Question:

Does the expressive vs. instrumental debate influence the discussion of voting systems?
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Discuss voting systems in an expressive framework

- Majority voting: Voter votes for a single candidate
- Approval voting: Voter picks an arbitrary subset of candidates
- Majority Judgment/Graded voting: Voter gives grades to candidates (1-10)
We

- present a formal Framework of Gilboa, Aragones and Weiss (2011) to compare approval and majority voting under expressive voting
- discuss this approach
- present an alternative framework

The framework

- Political debate consists of $n$-topics $T_1 \ldots T_n$.
- Stance on a topic is a number in $[-1 : 1]$
- every party $\vec{p}$ is a vector in $\{-1; 1\}^n$
- every voter $\vec{v}$ is a vector in $[-1, 1]^n$
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Relative weights, uncertainty
Majority Vote

Let $\mathcal{P}$ be the set of all parties. In **majority vote** each voter $v$ votes for the closest party. That is he minimizes

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- $\text{dist}$ is the euclidean distance
- The party with the most votes gets elected.
Approval voting

Let $\mathcal{P}$ be the set of all parties.
Approval voter: The position of a subset $I \subseteq \mathcal{P}$ is taken to be the straight average of its components:

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pos(I) := \frac{1}{|I|} \sum_{p \in I} p$$
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in approval voting each voter $\vec{v}$ approves of the coalition whose position is closest to his own:

$$\min_{I \subseteq \mathcal{P}} \text{dist}(\text{pos}(I), \vec{v})$$
Results of Aragones, Gilboa and Weiss

General Question: How much is required to motivate all voters to participate
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- In majority voting, the number of parties required to guarantee that everybody votes is exponential in the number of issues
- In approval voting 4 parties are enough to guarantee that everyone votes
- some stochastic results for number parties $= \text{number issues}$
Our critique

- Implicit coalition making highly improbable
  Equal weight assumption
  Discourse is shaped by single winner intuitions
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Our Approach

Approval voting: Evaluate parties individually

If a party \( p \) implements its policy the utility \( v \) gets on \( T \) is:

\[
|v_i| \text{ if } v_i \cdot p_i > 0
\]

Thus the utility \( v \) gets is:

\[
\sum_{\text{all}} |v_i| - 2 \sum_{\text{disagree}} |v_i| = \sum_{p_i} v_i\] 

\( v \) approves of \( p \) if \( \sum_{p_i} v_i \geq k \cdot \sum_{|v_i|} \)

For some threshold \( k \in (-1; 1] \). (Typically \( k \geq 0 \) )
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Geometric Interpretation

The algebraic definition is equivalent to: Accept a party $p$ if it is within an $\alpha$-cone round $\vec{v}$

\[ \alpha \text{ depends upon } n, k \text{ and } \vec{p}. \]
\[ \text{Holds } \arccos(k) \leq \alpha \leq \arccos\left(\frac{k}{\sqrt{n}}\right) \]
Justification of cone

- Reasoning about individual alternatives: Individual Criterion
- $\nu$ gives the relative weights of the different positions
- Cone represents level of satisfaction
Remark

Approval and Majority vote are compatible in the following sense: For any voter $\vec{v}$ and every party $\vec{p}$ holds:

$$p \text{ minimizes } dist(p, \vec{v}) \text{ iff } p \text{ maximizes } \frac{\vec{p} \cdot \vec{v}}{\sum |v_i|}$$
Results

For $k = 0$, i.e. $\alpha = 90^\circ$ we have exactly the same results as in Gilboa et al:

- 4 (resp $2n$) parties are enough to make everyone vote
- For fixed $\vec{v}$ and randomly chosen $n$ parties:
  \[ \lim_{n \to \infty} P(\exists \vec{p} | \vec{v} \text{ approves of } \vec{p}) = 1 \]
- For $k > 0$ exponentially many parties needed.
The critique reconsidered

- Plausible/in line with reasoning
The critique reconsidered

▶ Plausible/in line with reasoning ✓
The critique reconsidered

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- Does not facilitate the election of unfavourable parties
The critique reconsidered

- Plausible/in line with reasoning ✓
- Does not facilitate the election of unfavourable parties ✓
- Easily extendible to grade voting
Extension/Outlook: Focus Dynamics

- Focus of public attention changes over time
- Focus change has bigger impact on electoral outcome than opinion change
- Parties attempt to guide public focus to their areas of expertise
- Relative weights are not intrinsic
- Focus modelled by relative weights
Extension/Outlook: Focus Dynamics

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- Model every focus change as a vector $\vec{f} = (f_1 \ldots f_n) \in (0; 1)^n$. 

General Question: Which focus change should a party induce to maximize their electoral outcome?
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Focus change transforms voter \( \vec{v} = (v_1 \ldots v_n) \) into \( (f_1 \cdot v_1 \ldots f_n \cdot v_n) \).
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Wrap up

- Interplay between rational choiße theory and voting theory: Algebraic models as input
- Expressive voting changes discussion of voting systems
- Semantics for approval voting in line with natural intuitions
- Dynamic Aspects: Focus Change
Thank You