

# RETHINKING THE NEUTRALITY AXIOM IN JUDGMENT AGGREGATION

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**ABSTRACT**  
 How can we aggregate the judgments of a group of agents in a fair way? One solution is suggested by the popular neutrality axiom in judgment aggregation: if two judgments enjoy the same support amongst the agents, either both or neither of them should be part of the collective decision. This is a reasonable requirement in many scenarios, but we argue that for scenarios in which agents are asked to judge very diverse kinds of propositions, the classical neutrality axiom is much too strong. We thus propose a family of weaker neutrality axioms, parametrised by binary relations between the propositions.

## THE FRAMEWORK

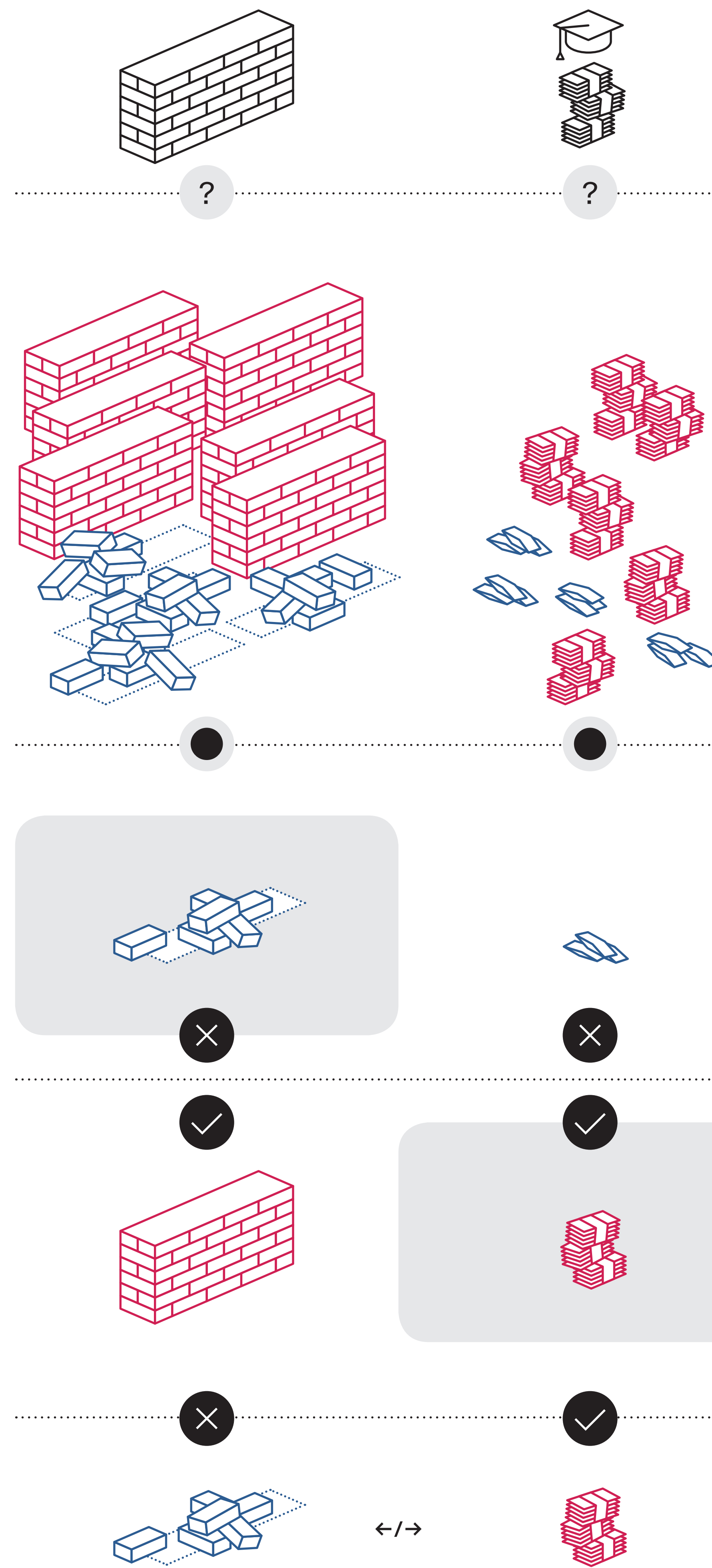
- An **agenda**  $\Phi$  contains all propositions on which a decision has to be made, modelled by formulas in *propositional logic*.
- An individual **judgment**  $J \subseteq \Phi$  is a (logically consistent) subset of the agenda.
- A **profile**  $\mathbf{J} = (J_1, \dots, J_n)$  represents the judgments of all agents  $1, \dots, n$ .
- $N_\phi$  is the set of agents who **agree** with proposition  $\phi$  in  $\mathbf{J}$ .
- A (resolute) **aggregation rule** is a function  $F$  that maps every possible profile  $\mathbf{J}$  to the group's judgment  $F(\mathbf{J}) \subseteq \Phi$ .
- $R \subseteq \Phi \times \Phi$  is a binary **relation** between the propositions in the agenda.

## NEUTRALITY

$$[N_\phi = N_\psi] \text{ implies that } [\phi \in F(\mathbf{J}) \Rightarrow \psi \in F(\mathbf{J})]$$

## RELATIONAL NEUTRALITY

$$[N_\phi = N_\psi \text{ and } \phi R \psi] \text{ implies that } [\phi \in F(\mathbf{J}) \Rightarrow \psi \in F(\mathbf{J})]$$



## A PROBLEM

**GOVERNOR:**  
 I am told to use neutral aggregation rules. Will it help me to decide this time?

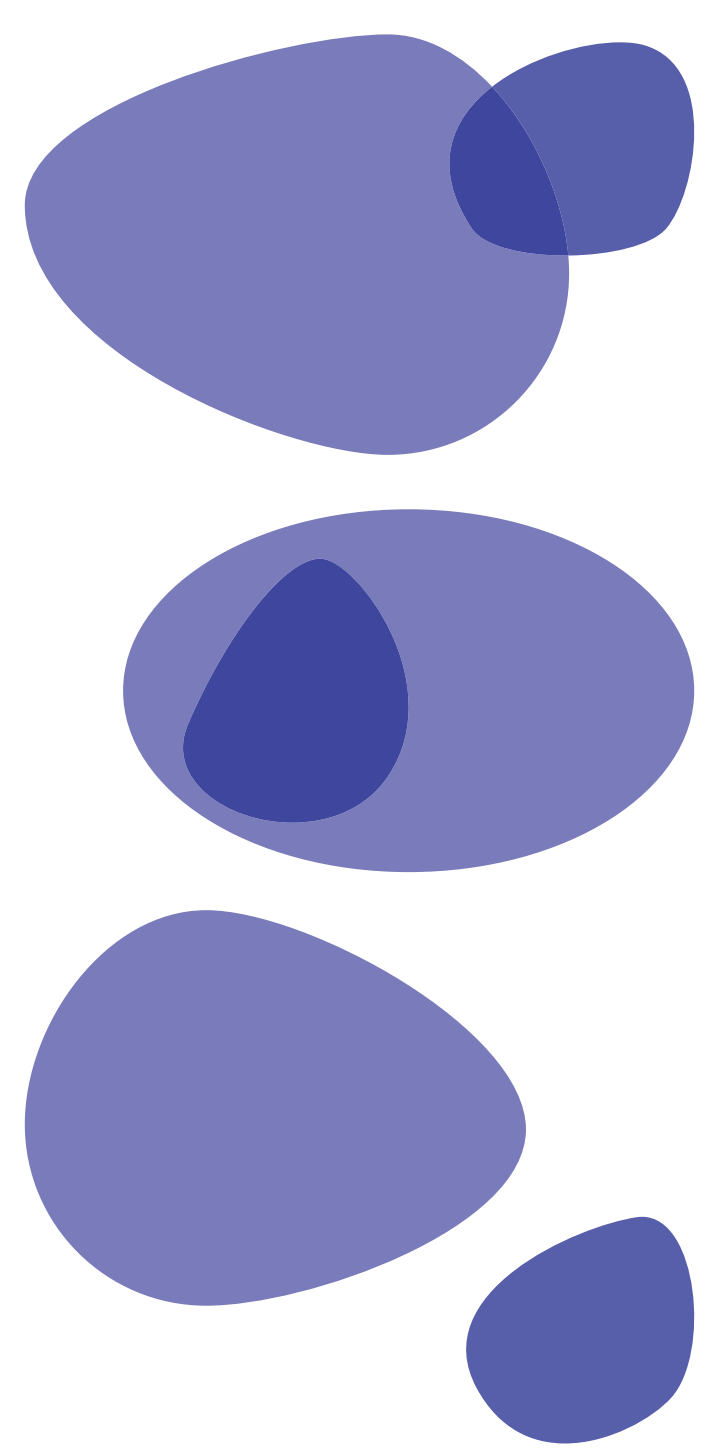
Although, did you know that the Kemeny rule is actually not neutral in judgment aggregation?

**GOVERNOR:**  
 60% is enough to increase tuition, but not to build a wall at the borders to another country!

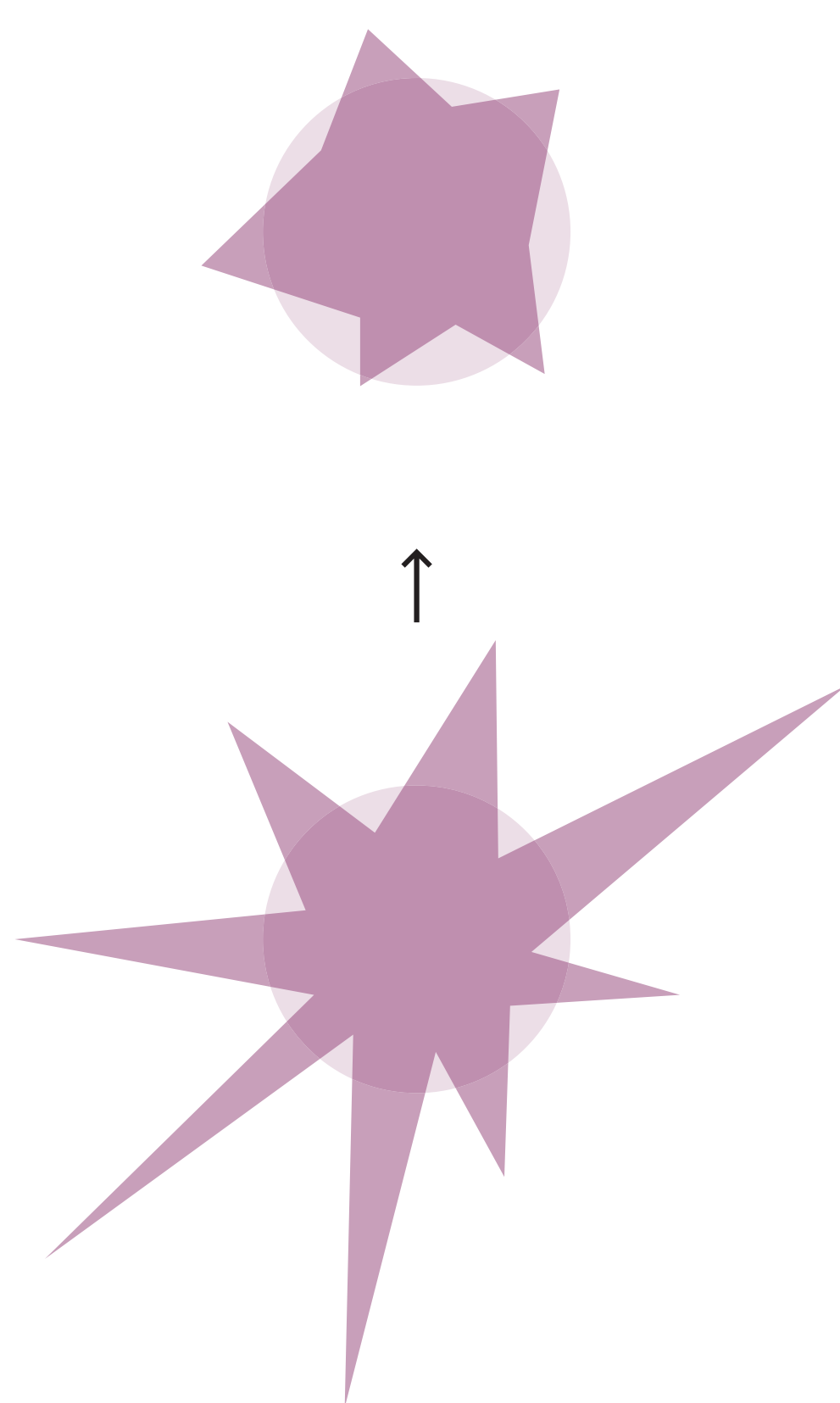
**GOVERNOR:**  
 Neutrality is too strong. Since the degree of the consequences of wall and tuition is different, maybe we need a not neutral rule.

## EXAMPLES

### NUMBER OF MODELS



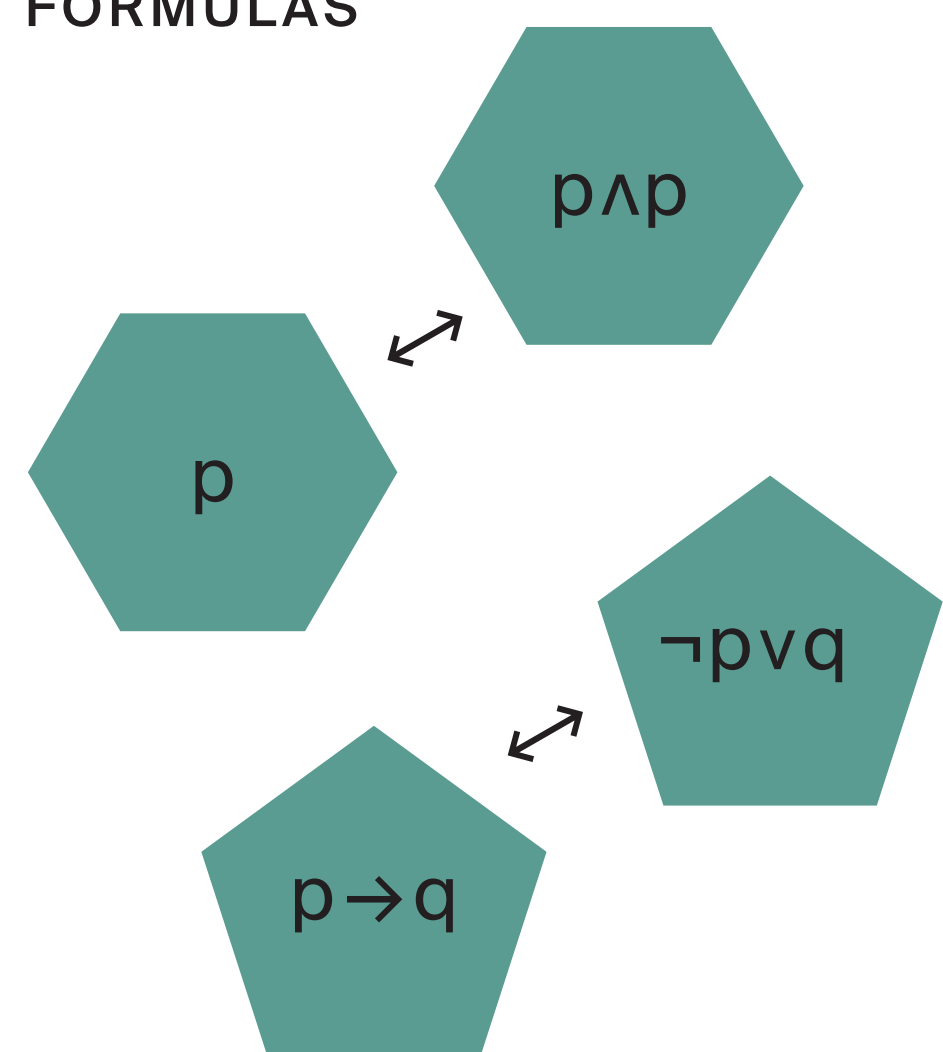
### LOW / HIGH IMPORTANCE



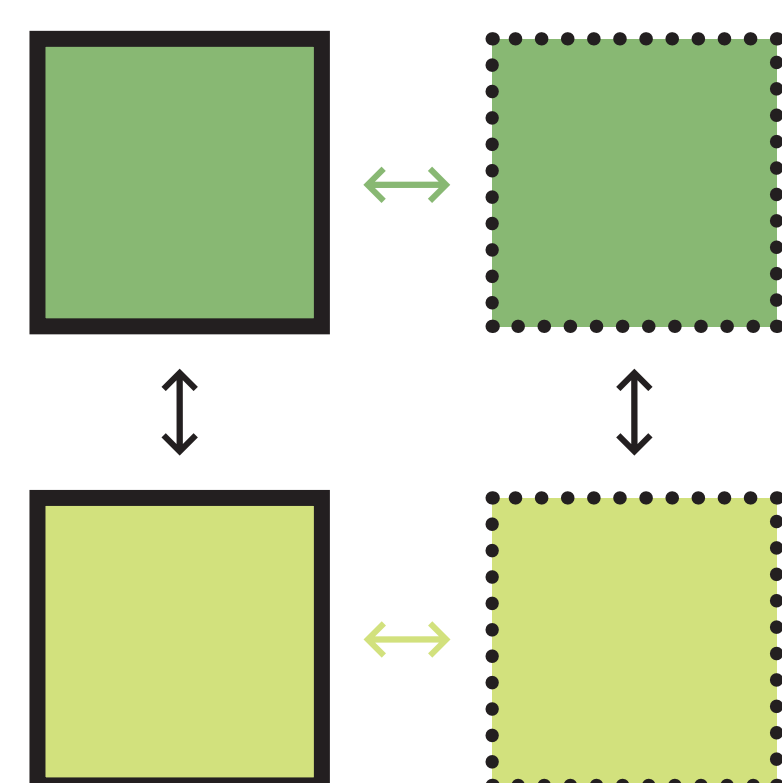
AGENT	BUILD WALL?	RAISE TUITION FEE?
AGENT 1	✓	✓
AGENT 2	✓	✓
AGENT 3	✓	✓
AGENT 4	✓	✓
AGENT 5	✓	✓
AGENT 6	✓	✓
AGENT 7	✗	✓
AGENT 8	✗	✗
AGENT 9	✗	✗
AGENT 10	✗	✗
<b>QUOTA</b>	66% (red circle)	51% (red circle)
	✗	✓

Quota rules assigning different quota to different propositions are not neutral, but can be relationally neutral.

### LOGICALLY EQUIVALENT FORMULAS



- PREMISES
- CONCLUSIONS
- SUBJECTIVE
- OBJECTIVE



### PREMISES

	p	q	r	z	→	c = p∧q∧r∧z	¬c
AGENT 1	✓	✗	✓	✓	⇒	✗	✓
AGENT 2	✓	✓	✗	✓	⇒	✗	✓
AGENT 3	✓	✓	✓	✗	⇒	✗	✓
	✓	✓	✓	✓	⇒	✓	?

### CONCLUSIONS

The premise-based rule is not neutral, but it is relationally neutral when all premises are related.

Also the Kemeny rule is relationally neutral if we relate propositions that are logically independent from each other.

$$\text{type } T \subseteq \Phi$$

$$R_T = \{(\phi, \psi) \mid \phi, \psi \in T\}$$