Representation Matters:
Characterisation and Impossibility Results for Interval Aggregation

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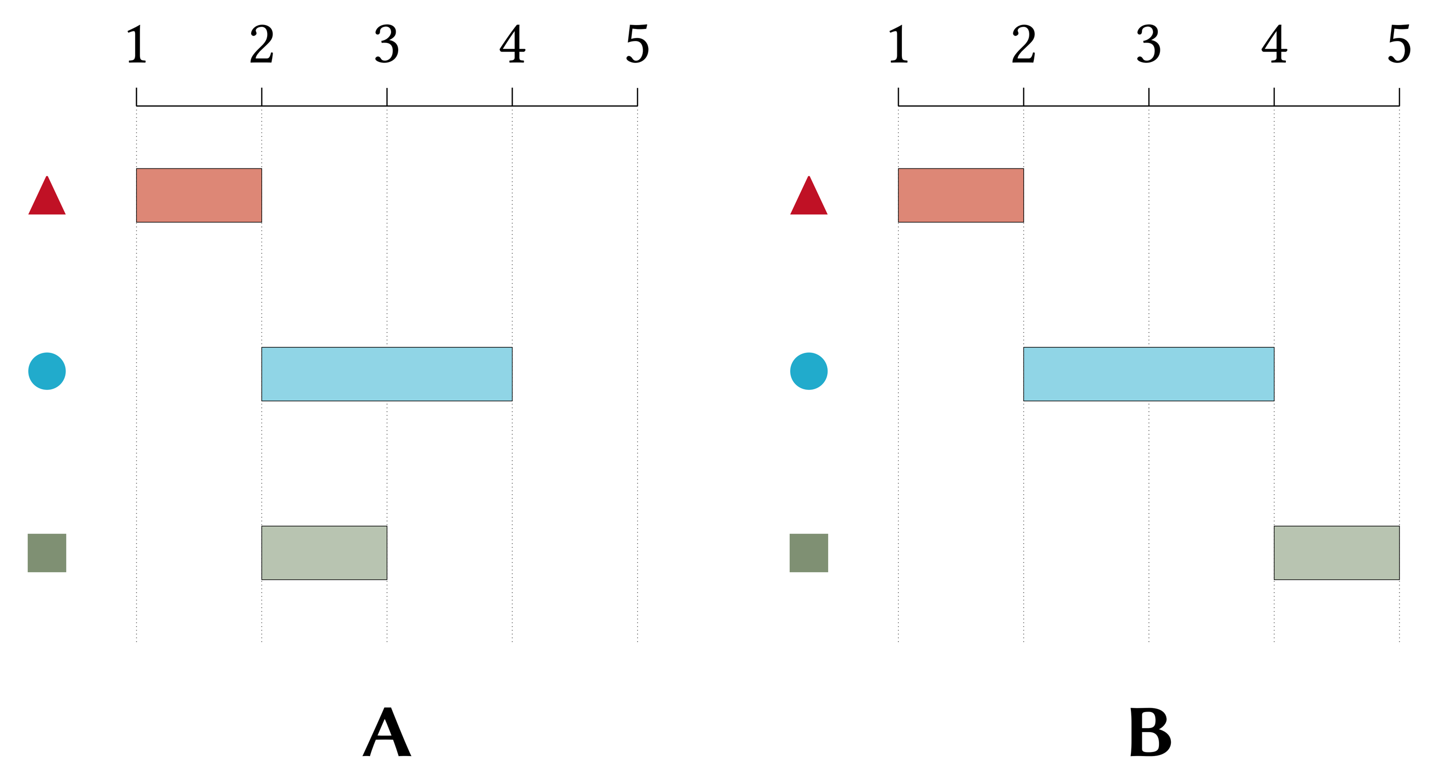
Each agent submits an interval on a given scale:
we want to aggregate them into a collective interval.



- The agents submit pieces of information about their intervals: let's aggregate them via the *median* rule.
- The *collective* (median) left endpoint in **A** and **B** is 2.

- Now, we ask for the right endpoints of their intervals, and use the median rule: what do we get for **A** and **B**?
- What if they instead submit the widths of their intervals?

Are there natural rules that can be represented both as aggregating separately the *left* and *right endpoints* and aggregating separately the *left endpoints* and the *widths*?

Scales, Intervals, Components

Scale: a nonempty $S \subseteq \mathbb{R}$, with a min and max element.

- $S = \{-3, 0, 2, 4, 7, 10, 12\}$ is a *discrete* scale
- $S' = [0, 1]$ is the *standard continuous* scale

Interval: a nonempty subset of the scale S with two extremes and *all* the points of S in-between those.

- $I = \{0, 2, 4\}$ is an interval of S, while $\{4, 10\}$ is not

Component: a function $\gamma : \mathcal{I}(S) \to D$, for a domain D.

- left endpoint (ℓ), right endpoint (r), width (w)

Representation-Faithfulness

Aggregation rule: function $F : \mathcal{I}(S)^n \to \mathcal{I}(S)$ from a profile of n agents' intervals on S to a collective interval.

F is faithful to a representation $\boldsymbol{\gamma}$ of q components, if there exist functions $f_k : D_k^n \to D_k$ for $k \in \{1, \ldots, q\}$ such that $F(\boldsymbol{I})$ for any profile $\boldsymbol{I}$ can be computed by applying each f_k to the corresponding component-profile.

F is a $\boldsymbol{\gamma}$-rule if it is faithful to $\boldsymbol{\gamma}$ and if $f_k(x, \ldots, x) = x$ for every $x \in D_k$ and for every $k \in \{1, \ldots, q\}$.

Impossibility theorem. For any given discrete scale S, every interval aggregation rule that is both an (ℓ, r)-rule and an (ℓ, w)-rule must be a **dictatorship**.

Characterisation theorem. For any given continuous scale S, a continuous interval aggregation rule is both an (ℓ, r)-rule and an (ℓ, w)-rule if and only if it is an (ℓ, r)-**weighted averaging rule**.