Sincerity and Manipulation under Approval Voting (Extended Abstract)

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Abstract

We formulate two new principles a voter might adopt when extending her preferences over candidates to preferences over sets of candidates and we show that, under approval voting, a voter who accepts those principles and who knows how the other voters will vote will never have an incentive to vote insincerely. We then discuss the consequences of this result for a number of standard principles of preference extension in view of sincere voting under approval voting.

Approval Voting

Under the system of *approval voting* (Brams and Fishburn, 2007) each voter can nominate any number of candidates, and the candidates receiving the most nominations win (if a unique winner is required this may have to be combined with a tie-breaking rule).

Let us assume that each voter is endowed with a *preference order* that is a ranking of the candidates standing for election. Under most other voting procedures, each voter submits a ranking of the candidates (rather than a subset of the candidates), and an important question in social choice theory is whether a voter has an incentive to vote *truthfully*, i.e., by submitting her true preference order as her ballot. For approval voting this is not a meaningful question: it is impossible to transmit one's true preference order by means of an approval ballot. Instead, we may ask under what circumstances a voter has an incentive to vote *sincerely*. An approval ballot is called *sincere* if the voter providing it ranks all approved candidates above all non-approved candidates.

Set Preferences

Approval voting may result in a set of tied winners. To evaluate the incentives of a voter when considering what ballot to cast we have to be able to make judgments about which of

two alternative sets of winning candidate she prefers. We do this by formulating principles for how a voter might extend her preferences over individual candidates to a preference order over sets of candidates. A large number of such principles has been discussed in the literature (Barberà et al., 2004).

The most basic such principle is the *Kelly Principle* (Kelly, 1977). It stipulates that a voter should prefer set A to set B if she prefers the candidate in A she likes the least to the candidate in B she likes the most. The Kelly Principle is uncontroversial and very weak. Here we propose two new axioms that may be imposed on top of this principle. The first is the *replacement axiom*.

> **Replacement.** If b is strictly preferred to a, then for any given set A at least one of the following three operations will result in a weakly preferred set: replacing a by b in A, removing a from A, or adding b to A.

This is a weaker version of an axiom proposed by Puppe (1995), who in turn reformulates a principle put forward by Sen (1991). Our second axiom is the *deletion axiom*.

> **Deletion.** If the set A is strictly preferred to the singleton containing only a, then removing a from A will result in a set that is weakly preferred to A.

This intuitively appealing axiom appears not to have been considered in the literature before.

Sincerity and Manipulation

We are interested in the following scenario: Suppose a voter has obtained information on what ballots the other voters are going to cast. Will she still vote sincerely or does she have an incentive to manipulate by means of an insincere ballot? We know that the answer is negative if the assumptions on preference extension we are willing to make are too weak: she may have an incentive to vote insincerely (Brams and Fishburn, 2007; Endriss, 2007). But under stronger assumptions we get a positive result:

> **Main Theorem.** Under approval voting, if a voter knows all other ballots and if she conforms to the Kelly Principle extended with both the replacement axiom and the deletion axiom, then she will not have an incentive to vote insincerely.

Corollaries to this theorem show that a voter also does not have an incentive to vote insincerely under several other assumptions, e.g., if she ranks sets according to their *median* elements, if her preferences over sets only depend on the *maximal* and *minimal* elements of those sets, and if she is an *expected-utility maximiser* who believes that ties will be broken using a uniform probability distribution.

These results complement earlier work in which we analysed the incentives to vote insincerely (i) under weaker assumptions on the principles for preference extension for elections with *small numbers of candidates* (Endriss, 2007), and (ii) under the additional assumption that a voter who cannot rank two sets of candidates based on a given principle of set extension will never resort to manipulation with respect to those sets (Endriss et al., 2009).

References

S. Barberà, W. Bossert, and P. Pattanaik. Ranking sets of objects. In *Handbook of Utility Theory*, volume 2. Kluwer Academic Publishers, 2004.

S. J. Brams and P. C. Fishburn. *Approval Voting*. Springer, 2nd edition, 2007.

U. Endriss. Vote manipulation in the presence of multiple sincere ballots. In D. Samet, editor, *Proceedings of the 11th Conference on Theoretical Aspects of Rationality and Knowledge (TARK-2007)*, pages 125–134. Presses Universitaires de Louvain, 2007.

U. Endriss, M. S. Pini, F. Rossi, and K. B. Venable. Preference aggregation over restricted ballot languages: Sincerity and strategy-proofness. In C. Boutilier, editor, *Proceedings of the 21st International Joint Conference on Artificial Intelligence (IJCAI-2009)*, pages 122–127, 2009.

J. Kelly. Strategy-proofness and social choice functions without single-valuedness. *Econometrica*, 45(2):439–446, 1977.

C. Puppe. Freedom of choice and rational decisions. *Social Choice and Welfare*, 12(2): 137–153, 1995.

A. K. Sen. Welfare, preference and freedom. *Journal of Econometrics*, 50(1–2):15–29, 1991.