Mixed Multi-unit Combinatorial Auctions for Supply Chain Management

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Mixed Multi-Unit Combinatorial Auctions (MMUCAs) allow agents to bid for bundles of goods to buy, goods to sell, and transformations of goods. In particular, MMUCAs offer a high potential to be employed for the automated assembly of supply chains of agents offering goods and services, and in general MMUCAs extend and generalise several types of combinatorial auctions. In this letter, we summarise our current research on MMUCAs and highlight lines for future research.

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1. INTRODUCTION

Supply chain formation (SCF) is a very promising application area for combinatorial auctions (CAs). For instance, Walsh et al. [2000] observe that production technologies often have to deal with strong complementarities: the inputs and outputs of a production process are strongly connected since a producer may risk to produce unsold goods, as well as fail to produce already sold goods when failing to obtain the inputs, thus losing credibility on the market. Hence, a supply chain can be regarded as an intricate network of producers (entities transforming input goods into output goods at a certain cost), and consumers interacting in a complex way. Nevertheless, the complementarities arising in SCF are different from the ones we do find in CAs. The complementarities in SCF arise because of the preconditions and postconditions of production processes: precedences and dependences along the supply chain must be taken into account. Hence, whilst in CAs the complementarities can be simply represented as relationships among goods, in SCF the complementarities involve not only goods, but also *transformation* (production) relationships along several levels of the supply chain.

In order to automate SCF, Walsh et al. [2000] have introduced the notion of

task dependency network (TDN) as a way of dealing with such complementarities. Nonetheless, although TDNs are indeed valuable to model SCF, it is our belief that further requirements (namely *expressiveness*, *computability*, and *formal analysis*) must be addressed to fully support automated SCF. As to *expressiveness requirements*, we must offer a fully expressive bidding language allowing to capture complementarities and substitutability among transformations and/or goods. As to *computational requirements*, we must ensure computational tractability of SCF while preserving optimality. Finally, as to *formal requirements*, we require a formalism that supports the formal study of structural and behavioural properties of a supply chain. Next, we outline our contributions along these directions and we discuss open issues for future research.

2. MIXED MULTI-UNIT COMBINATORIAL AUCTIONS

As a first contribution, in [Cerquides et al. 2007] we introduce mixed multi-unit combinatorial auctions (MMUCAs) as a generalisation of the standard model of combinatorial auctions. Rather than negotiating over simple goods, in MMUCAs the auctioneer and the bidders negotiate over transformations, each characterized by a set of input goods and a set of output goods. The intuitive meaning of a transformation is that the bidder offering the transformation is willing to produce the specified set of output goods after having received the specified input goods (and the payment specified in the bid). We provide a bidding language based on a set of operators that allow bidders to compose their bids in an intuitive way. Furthermore, we provide rewrite rules that allow us to translate any expression of the bidding language into an equivalent XOR-combination of atomic bids in an *XOR-language*, proven to be fully expressive. We also formally define the winner determination problem (WDP) for MMUCAs. In standard CAs, a solution to the WDP is a set of atomic bids to accept. In our setting, however, the *order* in which the auctioneer "uses" the accepted transformations matters. Thus, solving the WDP requires choosing the sequence in which the accepted bids should be implemented. In fact, the most important difference between MMUCAs and models known as double auctions or combinatorial exchanges is that these models do not have the concept of a *sequence of exchanges*, which is required if the intention is to model some sort of production process. Finally, we show how to map the WDP into an integer program (IP). While this provides a first algorithmic solution to the WDP, there is still room for improvement as the number of variables grows quadratically with the number of transformations mentioned in the bids.

3. FORMAL FRAMEWORK

Notice though that our main goal goes beyond solving the WDP. Thus, we are also concerned with providing a formal framework that serves to: (i) capture the optimisation problem faced by the auctioneer; (ii) analyse the computational properties of the winner determination problem; and (iii) facilitate the classification of WDPs, and hence provide guidance for developing efficient solution algorithms. We argue that our endeavour is necessary to formally characterise the WDP when transformations among goods hold and to subsequently found research on the WDP. In [Giovannucci et al. 2007] we propose *Weighted Petri Nets* as such a framework.

Petri nets are a graphical and mathematical modeling tool for discrete distributed systems that provide neat semantics to the notion of transformation. Furthermore, by borrowing results on Petri nets, we manage to dramatically reduce the number of decision variables involved in the optimisation problem posed by MMUCAs from quadratic to linear for a wide class of MMUCA WDPs.

4. TEST SUITE

Despite its potential for application, and unlike CAs, little is known about the practical application of MMUCAs since no empirical results have been reported on any WD algorithms. These results are unlikely to come up unless, along the lines of the research effort carried out in CAs, researchers are provided with algorithms or test suites to generate artificial data that is representative of the auction scenarios a WD algorithm is likely to encounter. Hence, WD algorithms could be accurately tested, compared, and improved. In [Vinyals and Cerquides 2007], we contribute to the practical application of MMUCAs along two directions. Firstly, we provide an algorithm to generate artificial data sets that are representative of the sort of scenarios a WD algorithm is likely to encounter. Secondly, we employ such an algorithm to generate artificial data and subsequently assess the performance of an Integer Programming (IP) implementation of a WD solver for MMUCA on CPLEX.

5. CURRENT AND FUTURE WORK

Nowadays, scalability is one of the most important problems for the real-world application of MMUCAs. Our current results indicate that the complexity of the WDP for problems involving a large number of transformations lies far beyond current computational resources. Thus, we envision two alternatives as future research lines: (i) the study of non-optimal, general purpose algorithms by means of local search; and (ii) the study of optimal algorithms that exploit the topological properties of the search space defined by the WDP. Furthermore, although MMUCAs consider the notion of *order* (the order in which transformations are implemented matters), they do not allow to explicitly express *time* constraints. Thus, we regard as necessary to extend the current bidding language as well as the WDP to support the explicit management of time. Finally, since we have conducted our research independently of any strategic issues bearing on mechanism design, the design of mechanisms for MMUCAs from a game-theoretic perspective is open.

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