Judgment Aggregation under Issue Dependencies

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Example: Choosing a Common Meal for a Party

A group of 23 gastro-entertainment professionals need to decide on the meal (1 dish + 1 drink) to be served at a party. What to choose?

	Chips?	*Beer?*	*Caviar?*	*Champagne?*
11 individuals:	Yes	Yes	No	No
10 individuals:	No	No	Yes	Yes
2 individuals:	No	Yes	Yes	No

Integrity Constraint: $(\mathit{Chips}\ \textsc{xor}\ \mathit{Caviar}) \wedge (\mathit{Beer}\ \textsc{xor}\ \mathit{Champagne})$

Talk Outline

- Binomial Rules: Issue Dependencies in Judgment Aggregation
- Theoretical Analysis: Axiomatics and Computational Complexity
- Experimental Analysis: Aggregating Hotel Reviews

Binomial Rules for Judgment Aggregation

Each *agent* accepts/rejects each *issue* (only some ballots are *rational*). An *aggregation rule* needs to map each *profile* to a consensus.

Idea: Award 1 point to potential outcome $B^\star$ for every ballot B_i and issue set I with $|I| \in K$ such that B_i and $B^\star$ fully agree on I.

$$F_K : \boldsymbol{B} \mapsto \underset{B^\star \text{ rational}}{\operatorname{argmax}} \sum_{B \in \boldsymbol{B}} \sum_{k \in K} \binom{\mathrm{Agr}(B, B^\star)}{k}$$

Most general definition also includes a *weight function* $w : K \to \mathbb{R}^+$.

Interesting special cases: $K = \{k\}$ (in which case w is irrelevant).
Note: this is *Kemeny rule* for $k = 1$ and *plurality-voter rule* for $k = m$.

Theoretical Results

Nice axiomatic properties (but full characterisation is open):

Theorem 1 *Binomial rules are amongst the very few rules discussed in the literature that satisfy both collective rationality and reinforcement:*

$$F(\boldsymbol{B}) \cap F(\boldsymbol{B'}) \neq \emptyset \;\; \textit{implies} \;\; F(\boldsymbol{B} \oplus \boldsymbol{B'}) = F(\boldsymbol{B}) \cap F(\boldsymbol{B'})$$

Binomial rules cover the range from the trivial to the highly intractable:

Theorem 2 *Winner determination for $F_{\{k\}}$ is in P if $(m-k) \in O(1)$.*

Theorem 3 *But the same problem is $\mathrm{P}^{\mathrm{NP}}[\log]$-complete if $k \in O(1)$.*

Experiment: Aggregating Hotel Reviews

Ratings for 6 features (*location*, etc.) of 1850 hotels from TripAdvisor.
Translation of 1–5 star scale: *accept* (4–5) or *reject* (1–3).

Results for the full data set not that interesting (see paper). But ...

Polarisation in Judgment Aggregation

In the paper, we develop a formal measure of *polarisation* of a profile, defined as the product of a *correlation* and an *uncertainty coefficient:*

- correlation = average strength of dependencies between issue pairs
- uncertainty = average disagreement on individual issues

A subset of *31 profiles* (opinions on 31 hotels) are "*highly polarised*".

The Compliant Reviewer Problem

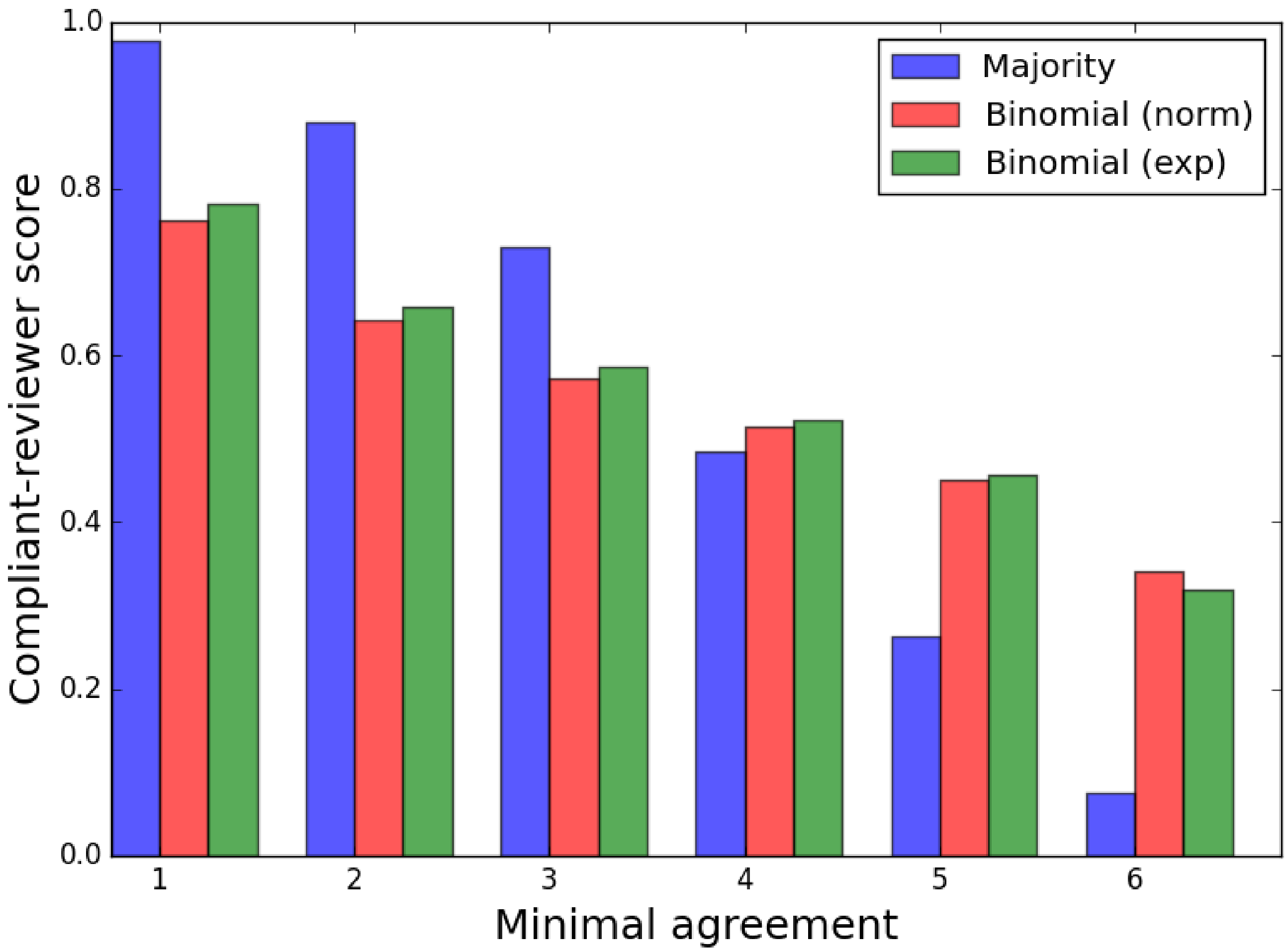
What makes for a good meta review (the result of the aggregation)?

> *You are writing a hotel review for an online magazine and you want to please as many of your readers as possible (to maximise the number of like's received). Suppose a reader will like your review if she agrees with you on $\geqslant k$ issues.*

We will use this *compliant-reviewer score* to evaluate our results.

Results for Highly Polarised Profiles

Comparing two instances of our family of rules with the majority rule.



Last Slide

Proposal for a *new family of judgment aggregation rules:*

- Attempt to account for hidden dependencies between issues
- Score agreement of outcome with ballots on subsets of issues
- Parameters: subset sizes to consider + weight function

Initial results for these so-called *binomial rules:*

- Includes *spectrum of rules* from Kemeny to plurality-voter rule
- Complexity: *winner determination* ranges from P to $\mathrm{P}^{\mathrm{NP}}[\log]$
- Axiomatics: both *collective rationality* and *reinforcement* ok
- Experiments: *good performance* for highly polarised hotel reviews

New concepts of potentially independent interest:

- Notion of *polarisation* of a profile in judgment aggregation
- *Compliant Reviewer Problem* to evaluate aggregation rules