Judgment Aggregation under Issue Dependencies

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[ joint work with Marco Costantini and Carla Groenland ]
Example: Choosing a Common Meal for a Party

A group of 23 gastro-entertainment professionals need to decide on the meal (1 dish + 1 drink) to be served at a party. What to choose?

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<tbody>
<tr>
<td>11 individuals:</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>10 individuals:</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>2 individuals:</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
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Integrity Constraint: \((\text{Chips} \text{xor} \text{Caviar}) \land (\text{Beer} \text{xor} \text{Champagne})\)
Talk Outline

- Binomial Rules: Issue Dependencies in Judgment Aggregation
- Theoretical Analysis: Axiomatics and Computational Complexity
- Experimental Analysis: Aggregating Hotel Reviews
Binomial Rules for Judgment Aggregation

Each agent accepts/rejects each issue (only some ballots are rational). An aggregation rule needs to map each profile to a consensus.

Idea: Award 1 point to potential outcome $B^*$ for every ballot $B_i$ and issue set $I$ with $|I| \in K$ such that $B_i$ and $B^*$ fully agree on $I$.

$$F_K : B \mapsto \arg \max_{B^* \text{ rational}} \sum_{B \in B} \sum_{k \in K} \binom{\text{Agr}(B, B^*)}{k}$$

Most general definition also includes a weight function $w : K \rightarrow \mathbb{R}^+$. Interesting special cases: $K = \{k\}$ (in which case $w$ is irrelevant).

Note: this is Kemeny rule for $k = 1$ and plurality-voter rule for $k = m$. 
Theoretical Results

Nice axiomatic properties (but full characterisation is open):

**Theorem 1** Binomial rules are amongst the very few rules discussed in the literature that satisfy both collective rationality and reinforcement:

\[ F(B) \cap F(B') \neq \emptyset \text{ implies } F(B \oplus B') = F(B) \cap F(B') \]

Binomial rules cover the range from the trivial to the highly intractable:

**Theorem 2** Winner determination for \( F\{k\} \) is in \( \mathbb{P} \) if \( (m - k) \in O(1) \).

**Theorem 3** But the same problem is \( \mathbb{P}^{\text{NP}[\log]} \)-complete if \( k \in O(1) \).
Experiment: Aggregating Hotel Reviews

Ratings for 6 features (location, etc.) of 1850 hotels from TripAdvisor. Translation of 1–5 star scale: accept (4–5) or reject (1–3).

Results for the full data set not that interesting (see paper). But . . .
Polarisation in Judgment Aggregation

In the paper, we develop a formal measure of *polarisation* of a profile, defined as the product of a *correlation* and an *uncertainty coefficient*:

- correlation = average strength of dependencies between issue pairs
- uncertainty = average disagreement on individual issues

A subset of 31 profiles (opinions on 31 hotels) are “highly polarised”.
The Compliant Reviewer Problem

What makes for a good meta review (the result of the aggregation)?

You are writing a hotel review for an online magazine and you want to please as many of your readers as possible (to maximise the number of like’s received). Suppose a reader will like your review if she agrees with you on $\geq k$ issues.

We will use this compliant-reviewer score to evaluate our results.
Results for Highly Polarised Profiles

Comparing two instances of our family of rules with the majority rule.

![Bar chart showing comparison of rules with majority rule for different levels of minimal agreement.]

- **Majority**
- **Binomial (norm)**
- **Binomial (exp)**
Last Slide

Proposal for a *new family of judgment aggregation rules*:

- Attempt to account for hidden dependencies between issues
- Score agreement of outcome with ballots on subsets of issues
- Parameters: subset sizes to consider + weight function

Initial results for these so-called *binomial rules*:

- Includes *spectrum of rules* from Kemeny to plurality-voter rule
- Complexity: winner determination ranges from $\text{P}$ to $\text{P}^{\text{NP}[\log]}$
- Axiomatics: both collective rationality and reinforcement ok
- Experiments: *good performance* for highly polarised hotel reviews

New concepts of potentially independent interest:

- Notion of *polarisation* of a profile in judgment aggregation
- *Compliant Reviewer Problem* to evaluate aggregation rules