# On the Communication Complexity of Multilateral Trading

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## What is this?

- We are interested in negotiation over resources in multiagent systems where agents may use very simple rationality criteria to decide on the acceptability of a proposed deal, but interaction patterns may be complex. In particular, *multilateral* deals (involving more than two agents) are possible.
- We are interested in the theoretical properties of such systems. This paper addresses the *communication complexity* of negotiating allocations of resources that are "socially optimal". That is, we focus on the length of negotiation processes and the amount of information exchanged, rather than on computational aspects.

## Talk Overview

- Our negotiation framework
  *rational but myopic agents trading discrete resources*
- Aspects of complexity
  *computational versus communication complexity*
- Some technical results for our negotiation framework
  *how many deals are required to reach an optimal allocation?*
- Conclusions

# Resource Allocation by Negotiation

- Finite set of *agents* $\mathcal{A}$ and finite set of discrete *resources* $\mathcal{R}$.
- An *allocation* $A$ is a partitioning of $\mathcal{R}$ amongst the agents in $\mathcal{A}$.
  Example: $A(i) = \{r_3, r_7\}$ — agent $i$ owns resources $r_3$ and $r_7$
- Every agent $i \in \mathcal{A}$ has got a *utility function* $u_i : 2^{\mathcal{R}} \rightarrow \mathbb{R}$.
  Example: $u_i(A) = u_i(A(i)) = 577.8$ — agent $i$ is pretty happy
- Agents may engage in negotiation to exchange resources in order to benefit either themselves or society as a whole.
- A *deal* $\delta = (A, A')$ is a pair of allocations (before/after).
- A deal may be accompanied by a payment to compensate some of the agents for a loss in utility. A *payment function* is a function $p : \mathcal{A} \rightarrow \mathbb{R}$ with $\sum_{i \in \mathcal{A}} p(i) = 0$.
  Example: $p(i) = 5$ and $p(j) = -5$ means that agent $i$ *pays* $\$5$, while agent $j$ *receives* $\$5$.

## The Local Perspective

A *rational* agent (who does not plan ahead) will only accept deals that improve its individual welfare:

**Definition 1** *A deal $\delta = (A, A')$ is called rational iff there exists a payment function $p$ such that $u_i(A') - u_i(A) > p(i)$ for all $i \in \mathcal{A}$, except possibly $p(i) = 0$ for agents $i$ with $A(i) = A'(i)$.*

## The Global Perspective

A *social welfare ordering* is a mapping from the preferences of the members of a society to a preference profile for society itself.

**Definition 2** *The (utilitarian) social welfare $sw(A)$ of an allocation of resources $A$ is defined as follows:*

$$sw(A) = \sum_{i \in \mathcal{A}} u_i(A)$$

## Linking the Local and the Global Perspective

From our AAMAS-2003 paper:

**Lemma 1** *A deal* $\delta = (A, A')$ *is rational iff* $sw(A) < sw(A')$.

Together with the finiteness of the allocation space this entails the following result on the feasibility of reaching an optimal allocation (due to Sandholm 1996/98; originally for distributed task allocation):

**Theorem 1** *Any sequence of rational deals will eventually result in an allocation of resources with maximal social welfare.*

▶ Question: How *complex* is the problem of finding such a socially optimal allocation of resources?

# Aspects of Complexity

(1) How many *deals* are required to reach an optimal allocation?

- communication complexity as number of individual deals
- technical results to follow

(2) How many *dialogue moves* are required to agree on one such deal?

- affects communication complexity as number of dialogue moves

(3) How expressive a *communication language* do we require?

- Minimum requirements: performatives *propose*, *accept*, *reject* + content language to specify multilateral deals
- affects communication complexity as number of bits exchanged

(4) How complex is the *reasoning* task faced by an agent when deciding on its next dialogue move?

- computational complexity (local rather than global view)

# Number of Deals

We have two results on *upper bounds* pertaining to the variant of our negotiation framework presented here (with side payments, general utility functions, and aiming at maximising utilitarian social welfare):

**Theorem 2 (Shortest path)** *A single rational deal is sufficient to reach an allocation with maximal social welfare.*

*Proof.* Use Lemma 1 [$\delta = (A, A')$ rational iff $sw(A) < sw(A')$]. □

**Theorem 3 (Longest path)** *A sequence of rational deals can consist of up to $|\mathcal{A}|^{|\mathcal{R}|} - 1$ deals, but not more.*

*Proof.* No allocation can be visited twice (same lemma) and there are $|\mathcal{A}|^{|\mathcal{R}|}$ distinct allocations $\Rightarrow$ upper bound follows ✓
To show that the upper bound is *tight*, we need to show that it is possible that all allocations have distinct social welfare (see paper). □

## More technical results are in the paper . . .

- Number of rational deals without side payments required to reach a Pareto optimal allocation of resources:
  - *Shortest path:* $\leq 1$
  - *Longest path:* $< |\mathcal{A}| \cdot (2^{|\mathcal{R}|} - 1)$
- Number of rational one-resource deals with side payments to reach an allocation with maximal social welfare in additive domains:
  - *Shortest path:* $\leq |\mathcal{R}|$
  - *Longest path:* $\leq |\mathcal{R}| \cdot (|\mathcal{A}| - 1)$
- Number of rational one-resource deals without side payments to reach an allocation with maximal social welfare in 0-1 domains:
  - *Shortest and longest path:* $\leq |\mathcal{R}|$

For all of the above, the *feasibility* of reaching the optimal allocation has been proved in our AAMAS-2003 paper.

# Conclusions

- We have proposed a distinction of different *aspects of complexity* of negotiating socially optimal allocations of resources:
  - communication complexity of reaching a socially optimal allocation (in terms of individual deals);
  - communication complexity of negotiating a single deal;
  - expressiveness of the agent communication language;
  - computational complexity of the next-move problem.
- We have given upper bounds on the *shortest* and the *longest path* to a socially optimal allocation for four variants of our framework.
- Future work: address the other three aspects of complexity; devise concrete negotiation protocols; analyse the complexity of multilateral trading for different notions of social optimality and different local acceptability criteria (*"welfare engineering"*); . . .