Judgment Aggregation with Rationality and Feasibility Constraints

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Example

The five members of a local government council have to decide on whether to approve funding for three community initiatives . . .

	School?	**Theatre?**	**Parking?**
Anita	0	0	1
Björn	1	1	1
Christina	1	0	1
Dolph	1	1	0
Zlatan	0	1	1
Majority	1	1	1

Rationality Constraint = "I should support at least one initiative"

Feasibility Constraint = "We cannot afford paying for all initiatives"

Talk Outline

I propose a new model of *judgment aggregation* that distinguishes between *rationality* (input) and *feasibility* (output) constraints. And:

- *Characterisation Theorem* (when does majority rule "work"?)
- Definition of *Majoritarian Aggregation Rules* (that always "work")
- Application: *Simulating Common Voting Rules*

The Model

The *agenda* is a set of propositions you may *accept* or *reject*.

A *judgment* is a function $J : \mathrm{Agenda} \to \{0, 1\}$.

An *aggregation rule* F maps any given *profile* $\boldsymbol{J} = (J_1, \ldots, J_n)$ of judgments, one for each of n *agents*, to a single compromise judgment.

Can describe *rationality* (input) and *feasibility* (output) *constraints* using propositonal logic. For $\mathrm{Agenda} = \{S, T, P\}$ we might use:

$$\textsc{Rat} = S \vee T \vee P \qquad \textsc{Feas} = \neg(S \wedge T \wedge P)$$

What we would like:

$$(J_1, \ldots, J_n) \in \mathrm{Mod}(\textsc{Rat})^n \quad \Longrightarrow \quad F(J_1, \ldots, J_n) \in \mathrm{Mod}(\textsc{Feas})$$

Characterisation Theorem for the Majority Rule

When can we use the majority rule without risking infeasible outcomes?

Need some terminology:

- A formula *simple* if it is equivalent to a conjunction of 2-clauses.
- The *prime implicates* of a formula are the logically strongest clauses that are entailed by that formula.

Theorem: *The majority rule guarantees feasible outcomes on all rational profiles iff these two conditions are satisfied:*

- *The feasibility constraint is entailed by the rationality constraint.*
- *Every nonsimple prime implicate of the feasibility constraint is entailed by a simple prime implicate of the rationality constraint.*

This generalises a seminal result by Nehring and Puppe (2007).

K. Nehring and C. Puppe. The Structure of Strategy-proof Social Choice. Part I: General Characterization and Possibility Results on Median Space. *JET*, 2007.

Majoritarian Aggregation Rules

The majority rule might return infeasible outcomes. So we need rules that "approximate" the ideal of the majority *and* guarantee feasibility:

$$\text{max-set}(\boldsymbol{J}, \textsc{Feas}) = \underset{J \in \text{Mod}(\textsc{Feas})}{\text{argsetmax}} \{\varphi \in \text{Agenda} : J(\varphi) = \text{Maj}(\boldsymbol{J})(\varphi)\}$$

$$\text{max-num}(\boldsymbol{J}, \textsc{Feas}) = \underset{J \in \text{Mod}(\textsc{Feas})}{\text{argmax}} |\{\varphi \in \text{Agenda} : J(\varphi) = \text{Maj}(\boldsymbol{J})(\varphi)\}|$$

$$\text{max-sum}(\boldsymbol{J}, \textsc{Feas}) = \underset{J \in \text{Mod}(\textsc{Feas})}{\text{argmax}} \sum_{i \in \text{Agents}} |\{\varphi \in \text{Agenda} : J(\varphi) = J_i(\varphi)\}|$$

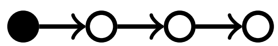
Simulating Common Voting Rules

While embedding preference aggregation is a basic staple in the JA literature, for many voting rules it has been difficult to simulate them.

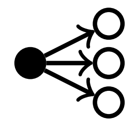
Refining an idea by Lang and Slavkovik (2013), we can do better.

Can speak about *preferences* by using agenda $\{p_{x \succcurlyeq y} \mid x, y \in \text{Alts}\}$.

Can express relevant *constraints*:

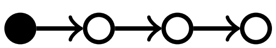
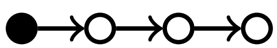
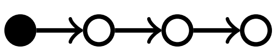
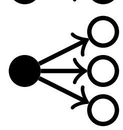
 $=$

$$\bigwedge_{x,y,z} (p_{x \succcurlyeq y} \wedge p_{y \succcurlyeq z} \to p_{x \succcurlyeq z}) \wedge \cdots$$

 $=$

$$\bigvee_{x} \bigwedge_{y \neq x} (p_{x \succcurlyeq y} \wedge \neg p_{y \succcurlyeq x}) \wedge \cdots$$

This yields the following *simulation results*:

Rationality	**Feasibility**	**max-set**	**max-num**	**max-sum**
		Top Cycle	Slater	Kemeny
		Uncovered Set	Copeland	Borda

J. Lang and M. Slavkovik. Judgment Aggreg. Rules and Voting Rules. ADT-2013.

Last Slide

What just happened:

- New model of JA that emphasises *rationality* and *feasibility*
- Feasibility of *majority rule:* characterisation via *prime implicates*
- Feasible *aggregation rules:* max-set, max-num, max-sum
- Convincing *embedding* of *Borda* voting rule (and others) into JA