Maximal Classes of Utility Functions for Efficient one-to-one Negotiation

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Introduction

- Multiagent systems may be thought of as “societies of agents”.
- Agents *negotiate* deals to exchange resources to benefit either themselves or society as a whole.
- Agents only use very simple rationality criteria to decide what deals to accept, but interaction patterns can be complex (*multilateral* deals).
- This talk: How can we restrict the range of utility functions agents may use to ensure that negotiation by means of structurally simple deals will result in optimal outcomes?
Resource Allocation by Negotiation

- Finite set of *agents* $A$ and finite set of indivisible *resources* $R$.
- An *allocation* $A$ is a partitioning of $R$ amongst the agents in $A$.
  
  Example: $A(i) = \{r_5, r_7\}$ — agent $i$ owns resources $r_5$ and $r_7$
- Every agent $i \in A$ has got a *utility function* $u_i : 2^R \rightarrow \mathbb{R}$.
  
  Example: $u_i(A) = u_i(A(i)) = 577.8$ — agent $i$ is pretty happy
- Agents may engage in negotiation to exchange resources in order to benefit either themselves or society as a whole.
- A *deal* $\delta = (A, A')$ is a pair of allocations (before/after).
- A deal may come with a number of side payments to compensate some of the agents for a loss in utility. A *payment function* is a function $p : A \rightarrow \mathbb{R}$ with $\sum_{i \in A} p(i) = 0$.
  
  Example: $p(i) = 5$ and $p(j) = -5$ means that agent $i$ *pays* €5, while agent $j$ *receives* €5.
Individual Rationality

A rational agent (who does not plan ahead) will only accept deals that improve its individual welfare:

- A deal $\delta = (A, A')$ is called individually rational iff there exists a payment function $p$ such that $u_i(A') - u_i(A) > p(i)$ for all $i \in A$, except possibly $p(i) = 0$ for agents $i$ with $A(i) = A'(i)$.

That is, an agent will only accept a deal iff it results in a gain in utility (or money) that strictly outweighs a possible loss in money (or utility).
Utilitarian Social Welfare

The *social welfare* associated with an allocation of resources $A$ is defined as follows:

$$sw(A) = \sum_{i \in Agents} u_i(A)$$

This is the so-called *utilitarian* definition of social welfare. Other definitions are possible (different story . . . ).
Example

Let $\mathcal{A} = \{ann, bob\}$ and $\mathcal{R} = \{chair, table\}$ and suppose our agents use the following utility functions:

\[
\begin{align*}
  u_{\text{ann}}(\emptyset) &= 0 & u_{\text{bob}}(\emptyset) &= 0 \\
  u_{\text{ann}}(\{\text{chair}\}) &= 2 & u_{\text{bob}}(\{\text{chair}\}) &= 3 \\
  u_{\text{ann}}(\{\text{table}\}) &= 3 & u_{\text{bob}}(\{\text{table}\}) &= 3 \\
  u_{\text{ann}}(\{\text{chair}, \text{table}\}) &= 7 & u_{\text{bob}}(\{\text{chair}, \text{table}\}) &= 8
\end{align*}
\]

Furthermore, suppose the initial allocation of resources is $A_0$ with $A_0(\text{ann}) = \{\text{chair, table}\}$ and $A_0(\text{bob}) = \emptyset$.

- Social welfare for allocation $A_0$ is 7, but it could be 8. By moving only a single resource from agent $\text{ann}$ to agent $\text{bob}$, the former would lose more than the latter would gain (not individually rational).

The only possible deal would be to move the whole set $\{\text{chair, table}\}$.
Linking the Local and the Global Perspectives

It turns out that individually rational deals are exactly those deals that increase social welfare:

**Lemma 1 (Rationality and social welfare)** A deal $\delta = (A, A')$ with side payments is individually rational iff $sw(A) < sw(A')$.

Even better, we obtain the following convergence result:

**Theorem 2 (Convergence)** Any sequence of individually rational deals will eventually result in an allocation with maximal social welfare.

However, optimal outcomes can only be guaranteed if the negotiation protocol allows for deals involving any number of agents and resources.

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Modular Domains

A utility function $u_i$ is called \textit{modular} iff it satisfies the following condition for all bundles $R_1, R_2 \subseteq \mathcal{R}$:

$$u_i(R_1 \cup R_2) = u_i(R_1) + u_i(R_2) - u_i(R_1 \cap R_2)$$

That is, in a modular domain there are no synergies between items; you can get the utility of a bundle by adding up the utilities of the items in that bundle.

Efficient negotiation in modular domains \textit{is} possible:

\textbf{Theorem 3 (Modular domains)} If all utility functions are modular, then individually rational 1-deals (involving just one resource) suffice to guarantee outcomes with maximal social welfare.
Sufficiency, Necessity, Maximality

- Theorem 3 says that the class of modular utility functions is sufficient for successful 1-deal negotiation.

- Is it also necessary? 
  Answer: No. It’s easy to construct examples (e.g. $u_i \equiv 10$).

- Is there any class of functions that is sufficient and necessary? 
  Answer: No. Seems surprising at first, but for a proof it suffices to find two sufficient classes the union of which is not sufficient.

- As there can be no unique class of utility functions characterising all situations where 1-deal negotiation works, we have looked for maximal classes . . .
Maximality of Modular Utilities

We say that a class of utility functions $\mathcal{F}$ permits 1-deal negotiation iff any sequence of individually rational 1-deals will converge to a socially optimal allocation whenever all utility functions belong to $\mathcal{F}$.

Another surprising result:

**Theorem 4 (Maximality)** Let $\mathcal{M}$ be the class of modular utility functions. Then for any class of utility functions $\mathcal{F}$ such that $\mathcal{M} \subset \mathcal{F}$, $\mathcal{F}$ does not permit 1-deal negotiation.

The proof is constructive: Given a non-modular utility function for one agent, it shows how to compose modular utilities for the other agents such that there is an initial allocation from which no optimal allocation can be reached by means of individually rational 1-deals alone.
Conclusions

• The class of *modular* utility functions is not only *sufficient* but also *maximal* for permitting agents to negotiate *socially optimal* allocations of resources by means of *rational 1-deals* alone.

• Many open problems and scope for new directions of research!
  Two concrete open questions related to this talk:
  – What other (interesting) classes of functions are maximal?
  – What classes of utility functions permit successful negotiation by means of rational bilateral deals alone?

• For more information on the field in general, have a look at the MARA (Multiagent Resource Allocation) website:
  
  \[ \text{http://www.illc.uva.nl/~ulle/MARA/} \]