Diversity of Preferences

Ulle Endriss
Institute for Logic, Language and Computation
University of Amsterdam

[ joint work with Vahid Hashemi ]
Preferences and Preference Profiles

Preferences are central ingredients for all questions of game theory. In social choice theory, we deal with profiles of preferences.

Agent 1: \( \triangle \succ ○ \succ □ \)
Agent 2: ○ \( \succ □ \succ △ \)
Agent 3: □ \( \succ △ \succ ○ \)
Agent 4: □ \( \succ △ \succ ○ \)
Agent 5: ○ \( \succ □ \succ △ \)

?
Preferences in Social Choice Research

Some issues with existing models of preferences:

• with many voters, $\sim 25\%$ of 5-alternative profiles have Condorcet cycles, but they play almost no role in real-world preference profiles

• theoretical complexity results in social choice (e.g., hardness of strategic manipulation) rely on very narrow base of examples

• domain restrictions (e.g., single-peakedness) often too restrictive

• unclear how to best generate preference data for experiments
Diversity of Preferences

Basic intuition:

The less diverse the preferences in a group of agents are, the easier it should be to come to mutually acceptable decisions.

Research agenda:

Make “preference diversity” precise and test this intuition!

Remainder of this Talk

- Formal framework, definition of the concept, examples
- Theoretical results: axiomatic method
- Experimental results: frequency of phenomena \textit{modulo} diversity
Formal Framework

Finite set of *alternatives* $\mathcal{X}$. A *preference* is a strict *linear order* on $\mathcal{X}$. $\mathcal{L}(\mathcal{X})$ is the set of all such preference orders.

Each of a finite set of *agents* $\mathcal{N} = \{1, \ldots, n\}$ expresses a preference, giving rise to a *profile* $\mathbf{R} = (R_1, \ldots, R_n) \in \mathcal{L}(\mathcal{X})^n$.

We propose the concept of *preference diversity index* (PDI) to make judgments about which of two profiles we consider more diverse:

A *PDI* is a function $\Delta : \mathcal{L}(\mathcal{X})^n \to \mathbb{R}^+ \cup \{0\}$, with $\Delta(\mathbf{R}) = 0$ for unanimous profiles $\mathbf{R} = (R, \ldots, R) \in \mathcal{L}(\mathcal{X})^n$. 
Examples for Specific PDI’s

Three options for defining the diversity of a given profile $R$:

- **Simple support-based PDI**: number of distinct preferences in $R$. Generalisation: count, for a given $k \leq m$, the number of distinct ordered $k$-tuples of alternatives appearing in $R$.

- **Distance-based PDI**: measure the distance (e.g., Kendall tau) between any two preferences in $R$ and then aggregate the values obtained (e.g., by computing their sum or their maximum).

- **Compromise-based PDI**: first aggregate the individual preferences (e.g., using the Borda rule), then compute the Kendall tau distance of each individual preference to that “compromise”, and finally aggregate (e.g., add) the values obtained.
Theoretical Results

Adopt the \textit{axiomatic method} from social choice theory to formulate and explore desirable properties of PDI’s . . .

- \textbf{Anonymity} (A): order of agents does not matter
- \textbf{Neutrality} (N): order of alternatives does not matter
- \textbf{Strong discernability}: no two equally diverse profiles (unless for A/N)
- \textbf{Weak discernability}: only unanimous profiles have diversity 0
- \textbf{Support invariance}: equally diverse if \( \{R_1, \ldots, R_n\} = \{R'_1, \ldots, R'_n\} \)
- \textbf{Independence}: \( R \succ R' \) implies \( R \oplus R \succ R' \oplus R \) for “new” \( R \)

Two results:

- \textbf{Proposition}: For \( n > m! \) and \( m > 2 \), there can be \textit{no} PDI that is both support-invariant and \textit{strongly discernable}.

- \textbf{Proposition}: A PDI is weakly discernible, support-invariant, and \textit{independent} if and only if it is the \textit{simple support-based PDI}. 
Experimental Results

Setup:
- distance-based PDI (Kendall tau, sum)
- 50 agents / 5 alternatives / 1M profiles
- $x$-axis: diversity (from 0 to max)

Right:
- Condorcet winners / cycles
- agreement between voting rules
- voter satisfaction

Bottom:
- $y$-axis: frequency of diversity $x$
- impartial culture vs. real data (AGH)
Last Slide

We have introduced a formal framework for studying the important concept of diversity of preferences in a group.

- **General** notion of PDI (preference diversity index)
- Three proposals for (families) of specific PDI’s
- **Axioms** for “good” PDI’s (impossibility / characterisation results)
- Experimental results: discern real from synthetic data
- Experimental results: choice-theoretic effects depend on diversity

Many opportunities for future research:

- More/better specific PDI’s to use in practice
- More/better axioms, more characterisation results
- Analytically prove facts about experimentally observed trends
- Use PDI’s to structure experimental work in preference handling