Applications of Logic in Social Choice Theory

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Outline

Points I'm going to try to make:

- *Social choice theory*, the formal study of mechanisms for collective decision making, is relevant to *multiagent systems*.
- *Logic* plays an important role in research in social choice theory.
- *This community* is well placed to make a contribution.

Things I'm going to talk about in support of these points:

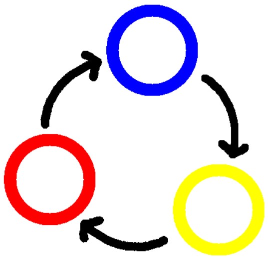
- Introduction to classical social choice theory
- Three research trends at the interface of logic and social choice:
  - Logics for social choice theory (+ automated reasoning)
  - Preferences and social choice in combinatorial domains
  - Judgment aggregation

Introduction to Social Choice Theory

Classic Example: The Condorcet Paradox

Social choice theory asks: how should we aggregate the preferences of the members of a group to obtain a "social preference"?

Expert 1: 🔵 $\succ$ 🟡 $\succ$ 🔴
Expert 2: 🟡 $\succ$ 🔴 $\succ$ 🔵
Expert 3: 🔴 $\succ$ 🔵 $\succ$ 🟡
Expert 4: 🔴 $\succ$ 🔵 $\succ$ 🟡
Expert 5: 🟡 $\succ$ 🔴 $\succ$ 🔵



Marie Jean Antoine Nicolas de Caritat (1743–1794), better known as the **Marquis de Condorcet**: Highly influential Mathematician, Philosopher, Political Scientist, Political Activist. Observed that the *majority rule* may produce inconsistent outcomes ("Condorcet Paradox").

Classic Result: Arrow's Impossibility Theorem

In 1951, K.J. Arrow published his famous *Impossibility Theorem:*

Any preference aggregation mechanism for *three* or more alternatives that satisfies the axioms of *Pareto* and *IIA* must be *dictatorial*.

- Pareto: if everyone says $X \succ Y$, then so should society.
- Independence of Irrelevant Alternatives (IIA): if society says $X \succ Y$ and someone changes her ranking of Z, then society should still say $X \succ Y$.

Kenneth J. Arrow (born 1921): American Economist; Professor Emeritus of Economics at Stanford; Nobel Prize in Economics 1972 (youngest recipient ever). His 1951 PhD thesis started modern Social Choice Theory. Google Scholar lists 10334 citations of the thesis.

Social Choice and AI (1)

Social choice theory has natural applications in AI:

- *Multiagent Systems:* to aggregate the beliefs + to coordinate the actions of groups of autonomous software agents
- *Search Engines:* to determine the most important sites based on links ("votes") + to aggregate the output of several search engines
- *Recommender Systems:* to recommend a product to a user based on earlier ratings by other users
- *AI Competitions:* to determine who has developed *the best* trading agent / SAT solver / RoboCup team

But not all of the classical assumptions will fit these new applications. So AI needs to develop *new models* and *ask new questions*.

Social Choice and AI (2)

Vice versa, techniques from AI, and computational techniques in general, are useful for advancing the state of the art in social choice:

- *Algorithms and Complexity:* to develop algorithms for (complex) voting procedures + to understand the hardness of "using" them
- *Knowledge Representation:* to compactly represent the preferences of individual agents over large spaces of alternatives
- *Logic and Automated Reasoning:* to formally model problems in social choice + to automatically verify (or discover) theorems

Indeed, you will find many papers on social choice at AI conferences (e.g., IJCAI, ECAI, AAAI, AAMAS) and many AI researchers participate in events dedicated to social choice (e.g., COMSOC).

Y. Chevaleyre, U. Endriss, J. Lang, and N. Maudet. A Short Introduction to Computational Social Choice. Proc. SOFSEM-2007.

Logics for Social Choice Theory

Logics for Social Choice Theory

The *axiomatic method* in SCT does borrow some terms from logic (“axiom”, “inconsistent”) and it will appeal to the logician, but it does not make any use of *logic* (no formal language, no inference rules).

We may want to develop *logics for social choice*. Reasons:

- Formalisation deepens *understanding*.
- Just as logic has been used to *verify* computer systems, we may want to try the same for social choice mechanisms.
- The *expressivity* needed to specify a property tells us something interesting about the underlying concept (e.g., do we need second-order quantification to speak about IIA?).

Modelling the Arrovian Framework

We can model Arrovian preference aggregation in *first-order logic*.

Important trick: introduce *situations* to refer to *profiles*. Examples:

- Transitivity: $\forall z, x_1, x_2, x_3, u.[I(z) \wedge A(x_1) \wedge A(x_2) \wedge A(x_3) \wedge S(u) \rightarrow (p(z, x_1, x_2, u) \wedge p(z, x_2, x_3, u) \rightarrow p(z, x_1, x_3, u))]$
- IIA: $\forall u_1, u_2, x, y.[S(u_1) \wedge S(u_2) \wedge A(x) \wedge A(y) \rightarrow [\forall z.(I(z) \rightarrow (p(z, x, y, u_1) \leftrightarrow p(z, x, y, u_2))) \rightarrow (w(x, y, u_1) \leftrightarrow w(x, y, u_2))]]$

Arrow's Theorem now reduces to this:

Theorem: $\mathrm{T}_{\mathrm{PA}} \cup \{\mathrm{PAR}, \mathrm{IIA}, \mathrm{NDIC}\}$ *has no finite model.*

Related work: (new) *modal logic* (Ågotnes et al., JAAMAS 2011); *propositional logic* (Tang & Lin, AIJ 2009); *HOL* (Nipkow, JAR 2009). For the latter two the focus is on automated reasoning.

U. Grandi and U. Endriss. First-Order Logic Formalisation of Arrow's Theorem. Proc. LORI-2009.

Research Challenges

What is the "right" logic to model social choice?

- don't fix the *set of individuals* (and alternatives) in the language
- model the *universal domain* assumption in an elegant manner
- support *automated reasoning*

How far can we push automation of reasoning about social choice?

- *full automation* *vs.* interactive theorem proving / ground instances
- *verification* of results in SCT and *discovery** of new theorems
- support *practical reasoning* about concrete mechanisms

*For a very simple area of SCT ("ranking sets of objects") we managed to achieve fully automated discovery of theorems.

C. Geist and U. Endriss. Automated Search for Impossibility Theorems in Social Choice Theory: Ranking Sets of Objects. *JAIR*, 40:143–174, 2011.

Preference Modelling and Social Choice in Combinatorial Domains

Example: The Paradox of Multiple Elections

13 voters are asked to each vote *yes* or *no* on three issues:

- 3 voters each vote for YNN, NYN, NNY.
- 1 voter each votes for YYY, YYN, YNY, NYY.
- No voter votes for NNN.

If we use the *plurality* rule *issue-by-issue*, then NNN wins, because on each issue 7 out of 13 vote *no*.

This is an instance of the *paradox of multiple elections:* the winning combination receives the fewest number of votes.

S.J. Brams, D.M. Kilgour, and W.S. Zwicker. The Paradox of Multiple Elections. *Social Choice and Welfare*, 15(2):211–236, 1998.

Social Choice in Combinatorial Domains

Many social choice problems have a *combinatorial structure:*

- During a *referendum* (in Switzerland, California, places like that), voters may be asked to vote on n different propositions.
- Elect a *committee* of k members from amongst n candidates.
- Find a good *allocation* of n indivisible goods to agents.

Seemingly small problems generate huge numbers of alternatives:

- Referendum on 10 propositions: $2^{10} = 1024$ possible outcomes
- Number of 3-member committees from 10 candidates: $\binom{10}{3} = 120$ (i.e., $120! \approx 6.7 \times 10^{198}$ possible rankings)
- Allocating 10 goods to 5 agents: $5^{10} = 9765625$ allocations and $2^{10} = 1024$ bundles for each agent to think about

Conclusion: We need good *languages* for representing preferences!

Compact Preference Representation

The most important language for COMSOC is that of CP-nets.

Also nice: logic-based languages with *prioritised* or *weighted goals*.

- Propositional language over PS. Want to model $u : 2^{PS} \to \mathbb{R}$.
- Formulas of $\mathcal{L}_{PS}$ represent goals. Weights represent importance.
- For each truth assignment, aggregate weights of satisfied formulas.

Results include:

- *Expressivity:* with sum aggregation, positive goals with positive weights can express all monotonic functions, and only those
- *Succinctness:* with sum aggregation, conjunctions of literals can express anything general formulas can, but do so less succinctly
- *Complexity:* with max aggregation, social welfare maximisation is NP-hard, even if all weighted goals have the form $(p \wedge q, 1)$

J. Uckelman. *More than the Sum of its Parts: Compact Preference Representation over Combinatorial Domains*. PhD thesis, ILLC, University of Amsterdam, 2009.

Combinatorial Vote: Example

Use the language of *prioritised goals* (1 has higher priority than 0) with *lexicographic aggregation* together with the *Borda rule:*

- Voter 1: $\{X{:}1,\ Y{:}0\}$ induces order $xy \succ_1 x\bar{y} \succ_1 \bar{x}y \succ_1 \bar{x}\bar{y}$
- Voter 2: $\{X \vee \neg Y{:}0\}$ induces order $x\bar{y} \sim_2 xy \sim_2 \bar{x}\bar{y} \succ_2 \bar{x}y$
- Voter 3: $\{\neg X{:}0,\ Y{:}0\}$ induces order $\bar{x}y \succ_3 \bar{x}\bar{y} \sim_3 xy \succ_3 x\bar{y}$

As the induced orders need not be strict linear orders, we use a *generalisation of the Borda rule:* an alternative gets as many points as she dominates other alternatives. So we get these Borda scores:

$$\begin{array}{ll} xy : 3+1+1 = 5 & \bar{x}y : 1+0+3 = 4 \\ x\bar{y} : 2+1+0 = 3 & \bar{x}\bar{y} : 0+1+1 = 2 \end{array}$$

So combinatorial alternative xy wins.

Combinatorial vote *proper* would be to compute the winner *directly* from the goal bases, without the detour via the induced orders.

Research Challenges

Develop approaches for social choice in combinatorial domains that are reasonably general and that balance *complexity* concerns and the need to limit *uncertainty* ($\leadsto$ paradoxes). This is wide open.

- *combinatorial vote:* develop algorithms for your favourite compact preference representation language and your favourite voting rule
- *sequential vote:* lift restrictive assumptions on voter preferences
- *new ideas* for different approaches

See our expository paper in the *AI Magazine* for an introduction.

Y. Chevaleyre, U. Endriss, J. Lang, and N. Maudet. Preference Handling in Combinatorial Domains: From AI to Social Choice. *AI Magazine*, 29(4):37–46, 2008.

Judgment Aggregation

Judgment Aggregation

Preferences are not the only structures we may wish to aggregate.
JA studies the aggregation of judgments on inter-related propositions.

	p	$p \to q$	q
Judge 1:	Yes	Yes	Yes
Judge 2:	No	Yes	No
Judge 3:	Yes	No	No
Majority:	Yes	Yes	No

Paradox: each *individual* judgment set is *consistent*, but the *collective* judgment arrived at using the *majority rule* is not

Research issues: impossibility theorems; characterisation of admissible agendas; proposals for "good" aggregation procedures; . . .

C. List and C. Puppe. Judgment Aggregation: A Survey. In *Handbook of Rational and Social Choice*. Oxford University Press, 2009.

Safety of the Agenda

One attempt to make JA more amenable to use in practice is our work on the *safety of the agenda* problem:

> An agenda Φ (set of formulas on which to vote) is *safe* for a set of axioms AX *iff* no aggregation procedure satisfying AX will ever "produce a paradox" when applied to Φ.

Results include:

- *Characterisation:* Φ is safe wrt. *anonymity* + *neutrality* *iff* any inconsistent subset of Φ has a subset $\{\varphi, \psi\}$ with $\models \varphi \leftrightarrow \neg\psi$
- *Complexity:* deciding SoA is Π_2^p-complete for many natural combinations of axioms

U. Endriss, U. Grandi, and D. Porello. Complexity of Judgment Aggregation: Safety of the Agenda. Proc. AAMAS-2010.

Research Challenges

Judgment aggregation seems obviously relevant to MAS and AI. But we still need to work out many of the details.

- will applying the JA methodology to logical frameworks richer than classical propositional logic yield interesting results?
- what axioms are relevant in what types of applications?
- no work on algorithms to date

Our paper on "ontology aggregation" tries to make a first step towards applying JA to Semantic Web issues.

D. Porello and U. Endriss. Ontology Merging as Social Choice. Proc. CLIMA-2011.

Last Slide

- COMSOC is an exciting area of research bringing together ideas from mathematical economics (particularly social choice theory) and computer science (including logic, AI, MAS, TCS).

  COMSOC Community: `http://www.illc.uva.nl/COMSOC/`

- In this talk, I focused on *three areas* of recent research activity:
  - Logics for Social Choice (and Automated Reasoning)
  - Preference and Social Choice in Combinatorial Domains
  - Judgment Aggregation

  Still lots to do!

- Extensive new *review paper* on Logic and SCT on my website.

U. Endriss. Logic and Social Choice Theory, 2011. Available from my website.