Voting with Incomplete Preferences

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Preview

I will present a model of *preference aggregation* in which preferences are *acyclic sets of pairwise comparisons* of alternatives and in which a voter's impact depends on the *number of comparisons* she provides.

Talk outline:

- Broader perspective: incompleteness in voting theory
- Model and motivation: aggregating sets of pairwise comparisons
- Axioms: restricted majoritarianism and splitting

Incompleteness in Voting: Research Directions

In voting theory, we usually model (true and reported) preferences as complete orders (weak or strict), defined on the set of alternatives.

But we should pay more attention to incompleteness:

- *Voters may have incomplete information regarding ballots of others.*

  Issues: informational barriers against strategic manipulation

- *The centre may have incomplete information regarding the ballots.*

  Issues: elicitation, possible winners, compiling intermediate results

- *Voters may have (reported) bona-fide incomplete preferences.*

  Issues: design and analysis of new voting rules (this talk)

Why Nonstandard Preferences?

Lots of scenarios where ballots (reported preferences) are *incomplete:*

- Voters only care about a subset of all pairwise comparisons.
- Voters can only reason about a subset of all pairwise comparisons.
- Voters are only asked about a subset of all pairwise comparisons.

We may even want to give up on *transitivity:*

> *You are asked to rank three apps: Facebook, Gmail, NYT. You prefer NYT to FB for the news. You prefer FB to Gmail for communication. Yet, you cannot rank NYT and Gmail.*

The Model

Sets of *voters* $N \subset \mathbb{N} = \{1, 2, \ldots\}$ and *alternatives* $A \subset \mathbb{A} = \{1, 2, \ldots\}$.

Every voter $i \in N$ provides a *ballot*, a set of pairwise comparisons:

$$R_i \quad = \quad \{\, (a, b) \in A \times A \mid a \text{ is ranked above } b \text{ by voter } i \,\}$$

$\mathcal{R}(A) \subseteq 2^{A \times A}$ is the the set of all such preference sets that are *acyclic*.

An *aggregation rule* returns one or more sets of pairwise comparisons for every given *profile of ballots* $\boldsymbol{R} = (R_1, \ldots, R_n) \in \mathcal{R}(A)^N$.

A Family of Aggregation Rules

We focus on a family of aggregation rules parametrised by a vector of *weights* $\boldsymbol{w} = (w_1, w_2, \ldots) \in \mathbb{R}^*_{>0}$ and a *type* $\mathcal{T}$ fixing $\mathcal{T}(A) \subseteq \mathcal{R}(A)$:

$$F^{\mathcal{T}}_{\boldsymbol{w}}(\boldsymbol{R}) \quad = \quad \underset{R \in \mathcal{T}(A)}{\text{argmax}} \sum_{i \in N} w_{|R_i|} \times |R_i \cap R|$$

Examples for *weight vectors* $\boldsymbol{w}$ of interest:

- constant: $(1, 1, 1, \ldots)$
- even-and-equal: $(1, \frac{1}{2}, \frac{1}{3}, \ldots)$
- lexicographic: $(\frac{1}{\Omega}, \frac{1}{\Omega^2}, \frac{1}{\Omega^3}, \ldots)$ for some large Ω
- almost constant: $(1+\frac{1}{\Omega}, 1+\frac{1}{\Omega^2}, 1+\frac{1}{\Omega^3}, \ldots)$ for some large Ω
- Discussion: $w_j < w_{j+1}$ unappealing? (incentive to make stuff up)

Examples for *output types* $\mathcal{T}$ of interest:

- linear type: $\mathcal{L}(A) = \{\, R \in \mathcal{R}(A) \mid R \text{ is a strict linear order} \,\}$
- winner type: $\mathcal{W}(A) = \{\, \{\, (a, b) \mid a \neq b \,\} \mid a \in A \,\}$

Special Case: Profiles of Linear Preferences

Suppose every voter provides a ballot that is a linear order: $R_i \in \mathcal{L}(A)$.

Observation: All ballots have the same size, so weights don't matter!

Proposition 1 $F^{\mathcal{L}}_{\boldsymbol{w}}(\boldsymbol{R}) = Kemeny(\boldsymbol{R})$ *for all* $\boldsymbol{R} \in \mathcal{L}(A)^N$ *and* $\boldsymbol{w}$.

Proposition 2 $top(F^{\mathcal{W}}_{\boldsymbol{w}}(\boldsymbol{R})) = Borda(\boldsymbol{R})$ *for all* $\boldsymbol{R} \in \mathcal{L}(A)^N$ *and* $\boldsymbol{w}$.

The Restricted Majority Principle

Respecting majority wishes is desirable but hard (Condorcet Paradox).

Idea: Restrict majoritarianism to obviously unproblematic pairs (a, b).

Call (a, b) *independent* of profile $\boldsymbol{R}$ if $(a, c), (c, a), (b, c), (c, b) \notin R_i$ for all other alternatives $c \in A \setminus \{a, b\}$ and all voters $i \in N$.

Axiom 1 *Rule F satisfies the restricted majority principle if, for every profile $\boldsymbol{R}$ and pair (a, b) that is independent of $\boldsymbol{R}$, this holds:*

$|\{ i \mid (a, b) \in R_i \}| > |\{ i \mid (b, a) \in R_i \}| \Rightarrow (b, a) \notin R$ *for all* $R \in F(\boldsymbol{R})$

A *constant-weight rule* is induced by $\boldsymbol{w} = (1, 1, 1, \ldots)$, among others.

Theorem 3 *The only rule $F_{\boldsymbol{w}}^{\mathcal{W}}$ (returning winners) that satisfies the restricted majority principle is the constant-weight rule (of type $\mathcal{W}$).*

Remark: We have analogous results for aggregation rules that return either *collective rankings* or *sets of winners*.

The Splitting Principle

Idea: Voters who care about disjoint matters should be able to form pre-election pacts w/o affecting the outcome ($\hookrightarrow$ vote trading).

Axiom 2 *Rule F satisfies the splitting principle if, for every profile $\boldsymbol{R}$ and group $S \subseteq N$ with $R_i \cap R_j = \emptyset$ and $|R_i| = |R_j|$ for all $i, j \in S$ as well as an acyclic $\bigcup_{j \in S} R_j$, this holds:*

$$F(\boldsymbol{R}) \;=\; F(\boldsymbol{R}') \text{ where } R'_i = \bigcup_{j \in S} R_j \text{ for } i \in S \text{ and } R'_i = R_i \text{ for } i \notin S$$

An *even-and-equal rule* is induced by $\boldsymbol{w} = (1, \frac{1}{2}, \frac{1}{3}, \ldots)$, among others.

Theorem 4 *The only rule $F^{\mathcal{W}}_{\boldsymbol{w}}$ (returning winners) that satisfies the splitting principle is the even-and-equal rule (of type $\mathcal{W}$).*

Remark: Analogous results for collective rankings and sets of winners.

Remark: Also works if the R_i in S are all required to be *singletons*. But we get an *impossibility result* if we drop the cardinality restriction.

Last Slide

I have argued that *incompleteness* is an important but understudied phenomenon in preference aggregation (and indeed SCT more broadly) and then presented a model of voting with incomplete preferences:

- Minimalist model of preferences: *acyclic pairwise comparisons*
- Flexibility by using *output types:* SWF, SCF, multiwinner, ...
- Characterisation results (*within* the space of weight rules) :
  - the *restricted majority principle* singles out weights $(1, 1, 1, \ldots)$
  - the *splitting principle* singles out weights $(1, \frac{1}{2}, \frac{1}{3}, \ldots)$

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