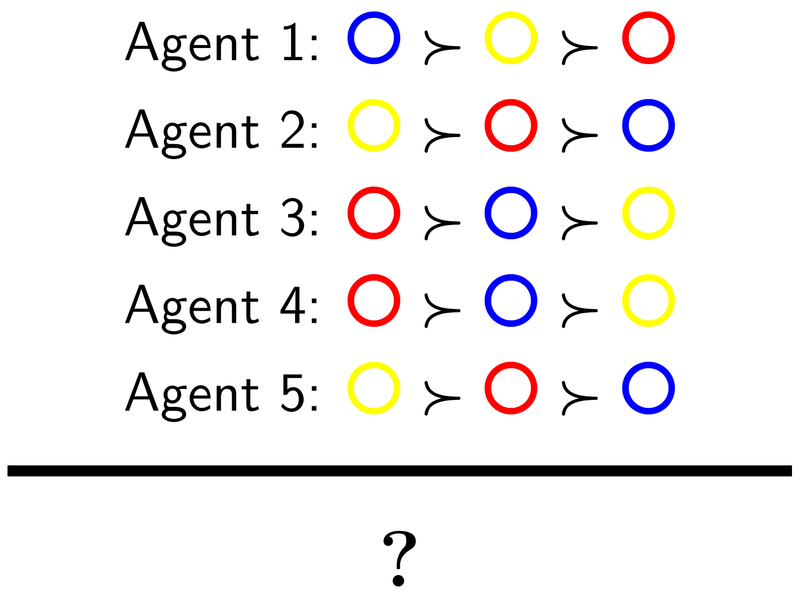
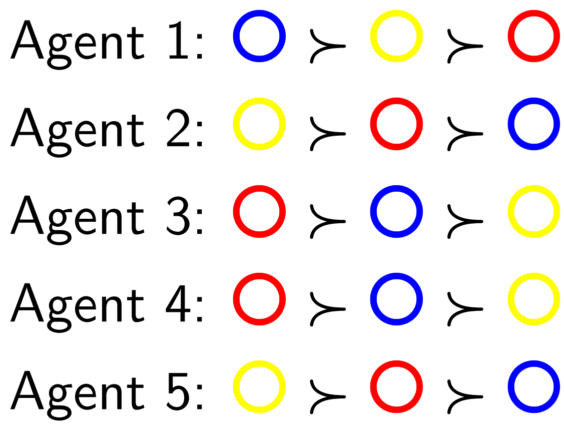
Making Collective Choices

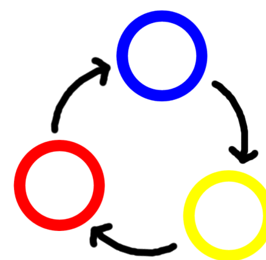
Ulle Endriss
Institute for Logic, Language and Computation
University of Amsterdam


Agent 1: ≻ ≻
Agent 2: ≻ ≻
Agent 3: ≻ ≻
Agent 4: ≻ ≻
Agent 5: ≻ ≻
?

Social Choice and the Condorcet Paradox

Social Choice Theory asks: how should we aggregate the preferences of the members of a group to obtain a "social preference"?





Marie Jean Antoine Nicolas de Caritat (1743–1794), better known as the **Marquis de Condorcet**: Highly influential Mathematician, Philosopher, Political Scientist, Political Activist. Observed that the *majority rule* may produce inconsistent outcomes ("Condorcet Paradox").

A Classic: Arrow's Impossibility Theorem

In 1951, K.J. Arrow published his famous *Impossibility Theorem:*

Any preference aggregation mechanism for *three* or more alternatives that satisfies the axioms of *unanimity* and *IIA* must be *dictatorial*.

- Unanimity: if everyone says $A \succ B$, then so should society.
- Independence of Irrelevant Alternatives (IIA): if society says $A \succ B$ and someone changes their ranking of C, then society should still say $A \succ B$.

Kenneth J. Arrow (born 1921): American Economist; Professor Emeritus of Economics at Stanford; Nobel Prize in Economics 1972 (youngest recipient ever). His 1951 PhD thesis started modern Social Choice Theory. Google Scholar lists 12,959 citations of the thesis.

Social Choice and Computer Science

Social choice theory has natural applications in computer science:

- *Search Engines:* to determine the most important sites based on links ("votes") + to aggregate the output of several search engines
- *Recommender Systems:* to recommend a product to a user based on earlier ratings by other users
- *Multiagent Systems:* to aggregate the beliefs + to coordinate the actions of groups of autonomous software agents

Vice versa, techniques from computer science are useful for advancing the state of the art in social choice theory ...

F. Brandt, V. Conitzer, and U. Endriss. Computational Social Choice. In G. Weiss (ed.), *Multiagent Systems*. MIT Press, 2013.

Outline of Rest of Talk

- More *examples* for challenges when making collective choices
- Identification of a *common pattern*
- Understanding links between properties of *aggregation rules* and properties of the *space of feasible choices*

	p	$p \rightarrow q$	q
Judge 1:	True	True	True
Judge 2:	True	False	False
Judge 3:	False	True	False

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	fund museum?	*fund school?*	*fund metro?*
Councillor 1:	Yes	Yes	No
Councillor 2:	Yes	No	Yes
Councillor 3:	No	Yes	Yes

?

[Constraint: we have money for *at most two projects*]

General Perspective

We can view many of our problems as problems of *binary aggregation:*

Do you rank option ○ above option ○ ? Yes/No

Do you believe formula "$p \to q$" is true? Yes/No

Do you want the new school to get funded? Yes/No

Each problem domain comes with its own *integrity constraints:*

Rankings should be transitive and not have any cycles.

The accepted set of formulas should be logically consistent.

We should fund at most two projects.

The *paradoxes* we have seen show that the *majority rule* does not *lift* our integrity constraints from the *individual* to the *collective* level.

Characterisation Results

So: Which aggregation rules lift which integrity constraints?

Classical perspective (following Arrow):

- specify formal *axioms* for intuitively "good" *aggregators*
  Examples: anonymity, neutrality, unanimity, monotonicity, ...

Our perspective:

- specify *expressive power* of language for *integrity constraints*
  Example: $\neg(\texttt{museum} \wedge \texttt{school} \wedge \texttt{metro})$

Example for a result:

Theorem 1 *An aggregator F will lift all integrity constraints that can be expressed as a conjunction of literals if and only if F is unanimous.*

U. Grandi and U. Endriss. Lifting Integrity Constraints in Binary Aggregation. *Artificial Intelligence*, 199–200:45–66, 2013.

Can we avoid all paradoxes?

<u>That is:</u> Are the aggregators that lift *all* integrity constraints? *Yes!*

Theorem 2 *An aggregator F will lift all integrity constraints if and only if F is a representative-voter rule (that is, if F is defined by a function g from profiles to agents via $F(B_1, \ldots, B_n) = B_{g(B_1,\ldots,B_n)}$).*

To be sure, this includes some pretty *bad* aggregators:

- Arrovian *dictatorships:* $g \equiv i$ (dictator fixed in advance)

But also some that look fairly *interesting:*

- return the individual ballot closest to the *majority* vector
- return the individual ballot closest to the *average* vector

U. Grandi and U. Endriss. Lifting Integrity Constraints in Binary Aggregation. *Artificial Intelligence*, 199–200:45–66, 2013.

Last Slide

I have tried to offer a glimpse at *computational social choice* and presented one particular line of research in this broad field:

- many paradoxes of collective choice have a common structure
- useful general model: *binary aggregation with integrity constraints*
- necessary and sufficient conditions for paradox-free aggregation

COMSOC is a booming field of research with lots of opportunities.

To find out more about the field, you could have a look at this website (biannual workshop series, PhD theses, mailing list):

`http://www.illc.uva.nl/COMSOC/`

U. Endriss. Computational Social Choice: Prospects and Challenges. *Procedia Computer Science*, 7:68–72, 2011.