Logic and Social Choice Theory

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Social Choice Theory

SCT studies collective decision making: how should we aggregate the preferences of the members of a group to obtain a "social preference"?

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$$\square \succ_2 \triangle \succ_2 \bigcirc$$

$$\bigcirc \succ_3 \square \succ_3 \triangle$$

$$?$$

SCT is traditionally studied in Economics and Political Science, but now also by "us": *Computational Social Choice*.

Talk Outline

- Computational Social Choice: *research area* and *community*
- LogiCCC Project “Computational Foundations of Social Choice”
- Three examples of ongoing research in Amsterdam:
  - Logical Modelling of Social Choice Problems
  - Compact Representation of Preferences
  - Judgment Aggregation

Computational Social Choice

Social choice theory studies mechanisms for *collective decision making*, such as voting procedures or protocols for fair division.

Computational social choice adds a computational perspective to this, and also explores the use of concepts from social choice in computing.

Examples:

- computational hardness as a barrier against strategic manipulation
- designing logics to model social mechanisms ("social software")
- coordination of multiagent systems through preference aggregation
- novel models of preference and aggregation (e.g., for AI)
- computational aspects of fair division

Y. Chevaleyre, U. Endriss, J. Lang, and N. Maudet. *A Short Introduction to Computational Social Choice*. Proc. SOFSEM-2007.

The COMSOC Research Community

- International Workshop on Computational Social Choice:
  - 1st edition: COMSOC-2006 in Amsterdam, December 2006
    48 paper submissions and 80 participants (14 countries)
  - 2nd edition; COMSOC-2008 in Liverpool, September 2008
    55 paper submissions and ∼80 participants (∼20 countries)
  - 3rd edition: COMSOC-2010 in Düsseldorf, September 2010
    Paper submission deadline will be in June 2010.
- Special issues in international journals:
  - *Mathematical Logic Quarterly*, vol. 55, no. 4, 2009
  - *Journal of Autonomous Agents and Multiagent Systems*, 2010
- Journals and conferences in AI, MAS, TCS, Logic, Econ, . . .
- COMSOC website: `http://www.illc.uva.nl/~ulle/COMSOC/`
  (workshop proceedings, related events, mailing list, etc.)

LogiCCC-CFSC: Aims and Objectives

Aim of the project:

- To develop sound foundations for the emerging field of *computational social choice*.

Key objectives:

- To deepen our understanding of complexity-theoretic and algorithmic issues arising in social choice theory.
(social choice and *theoretical computer science*)

- To develop logic-based languages for modeling and reasoning about social choice problems and preference structures.
(social choice and *logic*)

- To apply established techniques from AI, such as preference elicitation and learning, to problems of social choice.
(social choice and *artificial intelligence*)

LogiCCC-CFSC: Principal Investigators

- Felix Brandt (LMU University Munich, Germany)
- Ulle Endriss (University of Amsterdam, The Netherlands)
- Jeff Rosenschein (Hebrew University of Jerusalem, Israel)
- Jörg Rothe (University of Düsseldorf, Germany)
- Remzi Sanver (Istanbul Bilgi University, Turkey)

LogiCCC-CFSC: Associate Partners

- Vincent Conitzer (Duke University, USA)
- Edith Elkind (Nanyang Technological University, Singapore)
- Edith and Lane Hemaspaandra (RIT/Univ. of Rochester, USA)
- Jérôme Lang and Nicolas Maudet (University of Paris 9, France)
- Jean-François Laslier (Ecole Politechnique, France)

Arrow's Impossibility Theorem

It seems reasonable to require a *social welfare function* (SWF), mapping profiles of individual preference orderings to a social preference ordering, to satisfy the following axioms:

- *Unanimity* (UN): if every individual prefers alternative x over alternative y, then so should society
- *Independence of Irrelevant Alternatives* (IIA): social preference of x over y should only depend on individual pref's over x and y
- *Non-Dictatorship* (ND): no single individual should be able to impose a social preference ordering

Theorem 1 (Arrow, 1951) *For three or more alternatives, there exists no SWF that satisfies all of (UN), (IIA) and (ND).*

K.J. Arrow. *Social Choice and Individual Values*. 2nd edition, Wiley, 1963.

Full Formalisation of Arrow's Theorem

Logic has long been used to *formally specify* computer systems, facilitating formal or even *automatic verification* of various properties. Can we apply this methodology also to *social choice* mechanisms?

Tang and Lin (2009) show that the *"base case"* of Arrow's Theorem with 2 agents and 3 alternatives can be fully modelled in *propositional logic:*

- Automated theorem provers can verify $\textsc{Arrow}(2,3)$ to be correct in < 1 second — that's $(3!)^{3! \times 3!} \approx 10^{28}$ SWFs to check
- Opens up opportunities for quick sanity checks of hypotheses regarding new possibility and impossibility theorems.

Our ongoing work using *first-order logic* tries to go beyond such base cases.

P. Tang and F. Lin. Computer-aided Proofs of Arrow's and other Impossibility Theorems. *Artificial Intelligence*, 173(11):1041–1053, 2009

U. Grandi and U. Endriss. *First-Order Logic Formalisation of Arrow's Theorem*. Proc. LORI-2009. **[see talk on Thursday]**

Social Choice in Combinatorial Domains

Many social choice problems have a *combinatorial structure:*

- Elect a *committee* of k members from amongst n candidates.
- Find a fair *allocation* of n indivisible goods to agents.

Seemingly small problems generate huge numbers of alternatives:

- Number of 3-member committees from 10 candidates: $\binom{10}{3} = 120$ (i.e., $120! \approx 6.7 \times 10^{198}$ possible rankings)
- Allocating 10 goods to 5 agents: $5^{10} = 9765625$ allocations and $2^{10} = 1024$ bundles for each agent to think about

Conclusion: We need good *languages* for representing preferences!

Y. Chevaleyre, U. Endriss, J. Lang, and N. Maudet. Preference Handling in Combinatorial Domains: From AI to Social Choice. *AI Magazine*, Winter 2008.

Weighted Goals

A compact representation language for modelling utility functions (cardinal preferences) over products of binary domains —

Notation: finite set of propositional letters PS; propositional language $\mathcal{L}_{PS}$ over PS to describe requirements, e.g.:

$$p, \quad \neg p, \quad p \wedge q, \quad p \vee q$$

A *goalbase* is a set $G = \{(\varphi_i, \alpha_i)\}_i$ of pairs, each consisting of a (consistent) propositional formula $\varphi_i \in \mathcal{L}_{PS}$ and a real number α_i. The utility function u_G generated by G is defined by

$$u_G(M) \quad = \quad \sum\{\alpha_i \mid (\varphi_i, \alpha_i) \in G \text{ and } M \models \varphi_i\}$$

for all models $M \in 2^{PS}$. G is called the *generator* of u_G.

Different syntactic restrictions give different representation languages.

Some Results

Examples for our research on weighted goals:

- *Expressivity:* If all formulas and weights are positive, then we can express all monotonic utility function, and only those.
- *Succinctness:* Conjunctions of literals can express the same functions as general formulas, but do so strictly less succinctly.
- *Complexity:* Finding the most preferred model is NP-hard in general, but in $O(n \log n)$ if all formulas are literals.
- Applications: *combinatorial auctions* and *expressive voting*

J. Uckelman. *More than the Sum of its Parts: Compact Preference Representation over Combinatorial Domains*. PhD thesis, ILLC, University of Amsterdam, 2009.

J. Uckelman, Y. Chevaleyre, U. Endriss, and J. Lang. Representing Utility Functions via Weighted Goals. *Mathematical Logic Quarterly*, 55(4):341–361, 2009.

Finer Analysis via Linear Logic

Weighted goals *cannot* express statement such as this:

"getting p has value 5 to me, but getting p *twice* has value 8"

But this is important for *combinatorial auctions:*

- *Bidders* want to buy *bundles* of *goods* from an *auctioneer*.
- Bidders *bid* by reporting prices various bundles.
- *Winner determination:* find the *allocation* of goods to bidders that maximises the *revenue* for the auctioneer (sum of prices collected).
- Bidding is a form of *preference* representation: weighted goals can be used to encode bids, but only for *single-unit* CAs.

Resource-sensitive logics, in particular *linear logic*, can speak about the multiplicity of items. This idea is explored in our ongoing work.

D. Porello and U. Endriss. *Linear Logic for Bidding Languages*. Working Paper, ILLC, University of Amsterdam, 2009. **[see poster later today]**

Judgment Aggregation

Preferences are not the only structures that we may wish to aggregate. JA studies the aggregation of judgments on related propositions.

	p	$p \to q$	q
Agent 1:	Yes	Yes	Yes
Agent 2:	No	Yes	No
Agent 3:	Yes	No	No
Majority:	Yes	Yes	No

While each individual set of judgments is logically consistent, the collective judgment produced by the majority rule is not.

Research issues: impossibility theorems; characterisation of admissible agendas; proposals for "good" aggregation procedures; ...

C. List and C. Puppe. Judgment Aggregation: A Survey. *Handbook of Rational and Social Choice*. Oxford University Press, 2009.

Complexity of Judgment Aggregation

What about computational considerations in JA?

In ongoing work we address the following questions:

- *Safety of the Agenda:* Given an agenda Φ (set of propositions), can we guarantee that any aggregation procedure belonging to a given class of procedures (characterised via some axioms) will never "produce a paradox"?
- What is the computational complexity of deciding SoA?

U. Endriss, U. Grandi, and D. Porello. *Complexity of Judgment Aggregation*. Working Paper, ILLC, University of Amsterdam, 2009. **[see talk later today]**

Conclusion

- COMSOC is an exciting area of research bringing together ideas from mathematical economics (particularly social choice theory) and computer science (including logic).
- Three examples of ongoing research in Amsterdam:
  - Logical Modelling of Social Choice Problems
  - Compact Representation of Preferences
  - Judgment Aggregation
- COMSOC website: http://www.illc.uva.nl/~ulle/COMSOC/