Sincerity and Manipulation under Approval Voting

Ulle Endriss
Institute for Logic, Language and Computation
University of Amsterdam

Talk Outline + Approach

- *Voting Theory:* voters, candidates, preferences, voting rules, properties. Incentives to vote truthfully. Gibbard-Satterthwaite Theorem.

- *Approval Voting* (AV): ballot = *subset* of candidates
  But suppose preferences are *linear orders* $\Rightarrow$ cannot vote "*truthfully*".
  Ask instead for incentive to vote *sincerely* (approved $\dot{>}$ disapproved).

- Election may result in a tie $\Rightarrow$ *set of winners*.
  Need to *extend preferences* from individual to sets of candidates.
  Many standard principles for doing so: Kelly, Gärdenfors, ...

- Looking for *theorems* of this kind:

Under AV and assuming preference extension principle X, if you know all other ballots you'll never want to vote insincerely.

We prove this for a new, broad principle of preference extension and obtain it for several standard principles as a corollary.

Preference Extension

Suppose we know a voter's preferences over *elements* of a set $\mathcal{X}$.
What can we say about her preferences over *subsets* of $\mathcal{X}$?

<u>The model:</u>

- finite set of candidates $\mathcal{X}$
- total order $\dot{\geqslant}$ on $\mathcal{X}$ (preferences over candidates)
- weak order $\succcurlyeq$ on $2^{\mathcal{X}} \setminus \{\emptyset\}$ (preferences over nonempty sets)

A *preference extension principle* will fix *some* properties of $\succcurlyeq$ in terms of $\dot{\geqslant}$, but it (usually) does not determine $\succcurlyeq$ in its entirety.

Which principle to adopt depends on a voter's beliefs regarding the *tie-breaking rule* and her *attitude towards risk*.

S. Barberà, W. Bossert, and P. Pattanaik. Ranking sets of objects. In *Handbook of Utility Theory*, volume 2. Kluwer Academic Publishers, 2004. [survey paper]

Example: The Kelly Principle

This is the weakest of all standard principles. It states:

You will like set A more than set B ($A \succ B$) **if** you like any member of A more than any member of B ($a \dot{>} b$ for all $a \in A$ and $b \in B$).

More precisely, the Kelly Principle is defined by these three axioms:

$$\text{(EXT) } \{a\} \succ \{b\} \text{ if } a \dot{>} b$$

$$\text{(MAX) } \{\max(A)\} \succcurlyeq A \qquad \text{(MIN) } A \succcurlyeq \{\min(A)\}$$

For the Kelly Principle, it is *not* the case that a voter would never want to vote insincerely under approval voting. Example:

- Suppose $a \dot{>} b \dot{>} c$. Say, b got 10 approvals, and a and c 9 each.
- I can force one of these outcomes: $\{a, b, c\}$, $\{a, b\}$, $\{b\}$, $\{b, c\}$.
- KP forces $\{a, b\} \succcurlyeq \{b\} \succcurlyeq \{b, c\}$. And $\{a, b, c\} \succ \{a, b\}$ is *possible*.
- But I can only obtain $\{a, b, c\}$ by insincerely approving of a and c.

Goal

Suppose you have to vote in an election using approval voting.

Suppose you have obtained information on how the others will vote.

Will you (possibly) have an incentive to vote insincerely?

Under the Kelly Principle: *yes* (as we have seen)

What if we accept a *stronger* preference extension principle?

We would like to get this kind of result:

Under AV, a *fully informed* voter who conforms to *preference extension principle* X will always have a best response that is *sincere*.

Note: stronger extension principle $\Rightarrow$ better chance for a result

Another Example: Gärdenfors Principle

The *Gärdenfors Principle* is defined by these two axioms:

(GF1) $A \cup \{b\} \succ A$ if $b \dot{>} a$ for all $a \in A$

(GF2) $A \succ A \cup \{b\}$ if $a \dot{>} b$ for all $a \in A$

Possible interpretation: rational tie-breaker with unknown preferences

Fact: GP entails KP (ranks more pairs of sets).

We can show, by exhaustive enumeration (computer program):

Theorem 1 *Under AV for 3 candidates, a fully informed voter who conforms to the Gärdenfors Principle will always have a best response that is sincere.*

But for four candidates, there are again counterexamples (very few).

Yet Another Example: Pessimistic Voters

Call a voter *pessimistic* if she assumes tie-breaking will always pick the worst possible candidate form any given set:

(PES) $A \sim \{\min(A)\}$

This is a very strong principle.
We get a positive result with a simple proof:

Theorem 2 *Under AV, a fully informed voter who is pessimistic will always have a best response that is sincere.*

Proof: Consider the *pivotal* candidates (those with the most points before our would-be manipulator votes). Note that a nonempty subset of them must be amongst the winners.

Now consider this (sincere!) *strategy:* vote for your most preferred pivotal candidate and every candidate preferred to her. A pessimist cannot do better than that. ✓

Replacement Axiom

We may argue that is is reasonable to assume that a decision maker would be happy to replace a by b in set A, if she prefers b over a:

$$(A \setminus \{a\}) \cup \{b\} \succcurlyeq A \ \text{ if } \ b \mathrel{\dot{>}} a$$

This axiom has previously been used to characterise freedom of choice.

Weak Replacement Axiom

We will only require a weakened version of the replacement axiom:

(WRP) $(A \setminus \{a\}) \cup \{b\} \succcurlyeq A \ \text{ or } \ A \cup \{b\} \succcurlyeq A \ \text{ or } \ A \setminus \{a\} \succcurlyeq A \ \text{ if } \ b \mathrel{\dot{>}} a$

Deletion Axiom

We may argue that is is reasonable to assume that a decision maker would be happy to delete a from the set A if she likes a alone less than the full set A:

$$\text{(DEL) } A \setminus \{a\} \succcurlyeq A \;\; \text{if} \;\; A \succ \{a\}$$

This axiom seems not to have been proposed in the literature before.

Main Theorem

Theorem 3 *Under AV, a fully informed voter who conforms to the Kelly Principle, extended with both the weak replacement axiom and the deletion axiom, will always have a best response that is sincere.*

Discussion: How useful a result is this?

- Manipulation is a bad thing, because
  - ballots do not reflect will of the people
  - voters waste time computing their best ballot
- Should we accept Kelly + (WRP) + (DEL)?

Corollaries

For several natural preference extension principles, no voter who knows all other ballots will have an incentive to vote insincerely:

Theorem 4 *Any voter conforming to the Gärdenfors Principle* $+$ $A \sim \{\max(A), \min(A)\}$ *will always have a sincere best response.*

Theorem 5 *Any voter conforming to the Gärdenfors Principle* $+$ $A \sim \mathrm{med}(A)$ *will always have a best response that is sincere.*

Theorem 6 *An optimistic voter always has a sincere best response.*

Theorem 7 *Under uniform tie-breaking, any voter who is an expected utility maximiser will always have a sincere best response.*

Proof technique: show that the assumptions made here each entail the Kelly Principle as well as (WRP) and (DEL).

Last Slide

Common question in voting theory: will a voter vote truthfully?

For *approval voting*, we have to rephrase the question.

- *truthful* ballot: not a well-defined concept (set $\neq$ linear order)
- *sincere* ballot: more than one way to be sincere

So we ask: will a voter (knowing the other ballots) vote sincerely?

Answer: depends on the *preference extension principle*.

- Kelly Principle: no
- Other weak principles: no (but *almost* for small no. of candidates)
- Kelly + replacement + deletion: yes
- Various natural principles: yes

The *deletion axiom* may be of independent interest:

$$A \setminus \{a\} \succcurlyeq A \ \text{ if } \ A \succ \{a\}$$