Binary Aggregation with Integrity Constraints

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[ joint work with Umberto Grandi ]
Social Choice and the Condorcet Paradox

*Social Choice Theory* asks: how should we aggregate the preferences of the members of a group to obtain a “social preference”?

- Expert 1: $\bigcirc \succ \bigcirc \succ \bigcirc$
- Expert 2: $\bigcirc \succ \bigcirc \succ \bigcirc$
- Expert 3: $\bigcirc \succ \bigcirc \succ \bigcirc$
- Expert 4: $\bigcirc \succ \bigcirc \succ \bigcirc$
- Expert 5: $\bigcirc \succ \bigcirc \succ \bigcirc$

Marie Jean Antoine Nicolas de Caritat (1743–1794), better known as the **Marquis de Condorcet**: Highly influential Mathematician, Philosopher, Political Scientist, Political Activist. Observed that the *majority rule* may produce inconsistent outcomes (“Condorcet Paradox”).
A Classic: Arrow’s Impossibility Theorem

In 1951, K.J. Arrow published his famous *Impossibility Theorem*:

Any preference aggregation mechanism for *three* or more alternatives that satisfies the axioms of *unanimity* and *IIA* must be *dictatorial*.

- **Unanimity**: if everyone says $A \succ B$, then so should society.
- **Independence of Irrelevant Alternatives (IIA)**: if society says $A \succ B$ and someone changes their ranking of $C$, then society should still say $A \succ B$.

Social Choice and AI (1)

Social choice theory has natural applications in AI:

- **Search Engines**: to determine the most important sites based on links (“votes”) + to aggregate the output of several search engines
- **Recommender Systems**: to recommend a product to a user based on earlier ratings by other users
- **Multiagent Systems**: to aggregate the beliefs + to coordinate the actions of groups of autonomous software agents
- **AI Competitions**: to determine who has developed the best trading agent / SAT solver / RoboCup team

But not all of the classical assumptions will fit these new applications. So AI needs to develop new models and ask new questions.
Social Choice and AI (2)

Vice versa, techniques from AI, and computational techniques in general, are useful for advancing the state of the art in social choice:

- **Algorithms and Complexity**: to develop algorithms for (complex) voting procedures + to understand the hardness of “using” them

- **Knowledge Representation**: to compactly represent the preferences of individual agents over large spaces of alternatives

- **Logic and Automated Reasoning**: to formally model problems in social choice + to automatically verify (or discover) theorems

Indeed, you will find many papers on social choice at AI conferences (e.g., IJCAI, ECAI, AAAI, AAMAS) and many AI researchers participate in events dedicated to social choice (e.g., COMSOC).

Rest of this Talk

- Some more examples for paradoxes of aggregation
- General framework: *binary aggregation*
- New idea: lifting rationality assumptions
- Applications of that idea
Judgment Aggregation

\begin{align*}
  & p & p \rightarrow q & q \\
\text{Judge 1:} & \text{True} & \text{True} & \text{True} \\
\text{Judge 2:} & \text{True} & \text{False} & \text{False} \\
\text{Judge 3:} & \text{False} & \text{True} & \text{False} \\
\end{align*}

?
### Multiple Referenda

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<th>fund metro?</th>
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<td><strong>Voter 1:</strong></td>
<td>Yes</td>
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<td>No</td>
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<tr>
<td><strong>Voter 2:</strong></td>
<td>Yes</td>
<td>No</td>
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<tr>
<td><strong>Voter 3:</strong></td>
<td>No</td>
<td>Yes</td>
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\[ \text{Constraint: we have money for at most two projects} \]
General Perspective

The last example is actually pretty general. We can rephrase many aggregation problems as problems of *binary aggregation*:

- *Do you rank option ○ above option ○?* Yes/No
- *Do you believe formula “p → q” is true?* Yes/No
- *Do you want the new school to get funded?* Yes/No

Each problem domain comes with its own *rationality constraints*:

- *Rankings should be transitive and not have any cycles.*
- *The accepted set of formulas should be logically consistent.*
- *We should fund at most two projects.*

The *paradoxes* we have seen show that the *majority rule* does not *lift* our rationality constraints from the *individual* to the *collective* level.
Binary Aggregation with Integrity Constraints

Basic terminology and notation:

- Set of *individuals* $\mathcal{N} = \{1, \ldots, n\}$; set of *issues* $\mathcal{I} = \{1, \ldots, m\}$.
- Corresponding set of *propositional symbols* $PS = \{p_1, \ldots, p_m\}$ and *propositional language* $\mathcal{L}_{PS}$ interpreted on $D = \{0, 1\}^m$.
- An *aggregation procedure* is a function $F : D^\mathcal{N} \to D$. That is, each individual $i \in \mathcal{N}$ votes by submitting a *ballot* $B_i \in D$.
- An *integrity constraint* is a formula $IC \in \mathcal{L}_{PS}$ encoding a “rationality assumption”. Ballot $B \in D$ is *rational* iff $B \models IC$.

Now we can define our main concept:

- An aggregation procedure $F : D^\mathcal{N} \to D$ is *collectively rational* for $IC \in \mathcal{L}_{PS}$ if $B_i \models IC$ for all $i \in \mathcal{N}$ implies $F(B_1, \ldots, B_n) \models IC$. 

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Axioms for Binary Aggregation

Paradoxes show that aggregation is not trivial. We need to carefully formulate what we want, using so-called axioms.

- **Unanimity:** For any profile of rational ballots \((B_1, \ldots, B_n)\) and any \(x \in \{0, 1\}\), if \(b_{i,j} = x\) for all \(i \in \mathcal{N}\), then \(F(B_1, \ldots, B_n)_j = x\).

- **Anonymity:** For any rational profile \((B_1, \ldots, B_n)\) and any permutation \(\pi : \mathcal{N} \rightarrow \mathcal{N}\), we get \(F(B_1 \ldots B_n) = F(B_{\pi(1)} \ldots B_{\pi(n)})\).

- **Others:** neutrality, independence, monotonicity, \ldots

Axioms are (usually) defined for a given domain of aggregation: those profiles in \(\mathcal{D}^\mathcal{N}\) that are rational for a given IC.
Template for Results

Let $\mathcal{L} \subseteq \mathcal{L}_{PS}$ be a language of integrity constraints. By fixing $\mathcal{L}$ we fix a range of possible domains of aggregation.

Two ways of defining classes of aggregation procedures:

- The class of procedures defined by a given list of axioms $AX$:
  $\mathcal{F}_\mathcal{L}[AX] := \{ F : D^N \to D \mid F \text{ satisfies } AX \text{ on all } \mathcal{L}-\text{domains} \}$

- The class of procedures that lift all integrity constraints in $\mathcal{L}$:
  $\mathcal{CR}[\mathcal{L}] := \{ F : D^N \to D \mid F \text{ is collect. rat. for all } IC \in \mathcal{L} \}$

What we want:
$\mathcal{CR}[\mathcal{L}] = \mathcal{F}_\mathcal{L}[AX]$
Example for a Characterisation Result

Cubes (= conjunctions of literals) are lifted by an aggregation procedure \textit{iff} that procedure satisfies \textit{unanimity}:

\[\mathcal{CR}[\text{cubes}] = \mathcal{F}_{\text{cubes}}[\text{Unanimity}]\]

For more results of this sort, see the paper cited below.

Application: Good Binary Aggregation Procedures

Is there a procedure that will lift every integrity constraint? Yes!

\[ F \] will lift every \( IC \in \mathcal{L}_{PS} \) iff \( F \) is a generalised dictatorship, i.e., iff there exists a function \( g : \mathcal{D}^N \rightarrow \mathcal{N} \) such that always
\[ F(B_1, \ldots, B_n) = B_{g(B_1, \ldots, B_n)}. \]

The class of generalised dictatorships includes:

- proper dictatorship \( F_i : (B_1, \ldots, B_n) \mapsto B_i \) for fixed \( i \in \mathcal{N} \)
- distance-based generalised dictatorships mapping \( (B_1, \ldots, B_n) \) to that \( B_i \) that minimises the sum of the Hamming distances to the others (+ tie-breaking). An attractive procedure!

More applications are discussed in the paper cited below.

Binary aggregation with integrity constraints:

- *language* to express *rationality assumptions* in binary aggregation
- concept of *collective rationality* with respect to an IC
- characterisation results, relating *axioms* and *languages*
- *applications*: [preference + judgment aggreg.], good procedures

Bigger picture:

- *Axiomatic Method* in SCT: derive sophisticated result for specific domain (with specific rationality assumptions) and specific axioms
- “AI Approach”: need machinery to reason about many different application-specific domains, rationality assumptions, and axioms

Broader research area:

- Computational Social Choice, see [www.illc.uva.nl/COMSOC/](http://www.illc.uva.nl/COMSOC/)