# Binary Aggregation with Integrity Constraints

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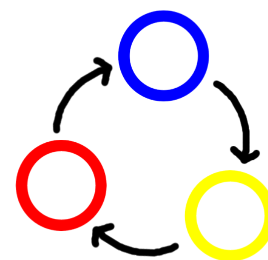
[ joint work with Umberto Grandi ]

# Social Choice and the Condorcet Paradox

*Social Choice Theory* asks: how should we aggregate the preferences of the members of a group to obtain a "social preference"?

Expert 1: 🔵 $\succ$ 🟡 $\succ$ 🔴
Expert 2: 🟡 $\succ$ 🔴 $\succ$ 🔵
Expert 3: 🔴 $\succ$ 🔵 $\succ$ 🟡
Expert 4: 🔴 $\succ$ 🔵 $\succ$ 🟡
Expert 5: 🟡 $\succ$ 🔴 $\succ$ 🔵



Marie Jean Antoine Nicolas de Caritat (1743–1794), better known as the **Marquis de Condorcet**: Highly influential Mathematician, Philosopher, Political Scientist, Political Activist. Observed that the *majority rule* may produce inconsistent outcomes ("Condorcet Paradox").

# A Classic: Arrow's Impossibility Theorem

In 1951, K.J. Arrow published his famous *Impossibility Theorem:*

Any preference aggregation mechanism for *three* or more alternatives that satisfies the axioms of *unanimity* and *IIA* must be *dictatorial*.

- Unanimity: if everyone says $A \succ B$, then so should society.
- Independence of Irrelevant Alternatives (IIA): if society says $A \succ B$ and someone changes their ranking of $C$, then society should still say $A \succ B$.

**Kenneth J. Arrow** (born 1921): American Economist; Professor Emeritus of Economics at Stanford; Nobel Prize in Economics 1972 (youngest recipient ever). His 1951 PhD thesis started modern Social Choice Theory. Google Scholar lists 11,828 citations of the thesis.

# Social Choice and AI (1)

Social choice theory has natural applications in AI:

- *Search Engines:* to determine the most important sites based on links ("votes") + to aggregate the output of several search engines
- *Recommender Systems:* to recommend a product to a user based on earlier ratings by other users
- *Multiagent Systems:* to aggregate the beliefs + to coordinate the actions of groups of autonomous software agents
- *AI Competitions:* to determine who has developed *the best* trading agent / SAT solver / RoboCup team

But not all of the classical assumptions will fit these new applications. So AI needs to develop *new models* and *ask new questions*.

## Social Choice and AI (2)

*Vice versa*, techniques from AI, and computational techniques in general, are useful for advancing the state of the art in social choice:

- *Algorithms and Complexity:* to develop algorithms for (complex) voting procedures + to understand the hardness of "using" them
- *Knowledge Representation:* to compactly represent the preferences of individual agents over large spaces of alternatives
- *Logic and Automated Reasoning:* to formally model problems in social choice + to automatically verify (or discover) theorems

Indeed, you will find many papers on social choice at AI conferences (e.g., IJCAI, ECAI, AAAI, AAMAS) and many AI researchers participate in events dedicated to social choice (e.g., COMSOC).

F. Brandt, V. Conitzer, and U. Endriss. Computational Social Choice. In G. Weiss (ed.), *Multiagent Systems*. MIT Press. To appear in 2012.

## Rest of this Talk

- Some more examples for paradoxes of aggregation
- General framework: *binary aggregation*
- New idea: lifting rationality assumptions
- Applications of that idea

## Judgment Aggregation

| | $p$ | $p \to q$ | $q$ |
|---|---|---|---|
| **Judge 1:** | True | True | True |
| **Judge 2:** | True | False | False |
| **Judge 3:** | False | True | False |
| | | ? | |

## Multiple Referenda

| | *fund museum?* | *fund school?* | *fund metro?* |
|---|---|---|---|
| **Voter 1:** | Yes | Yes | No |
| **Voter 2:** | Yes | No | Yes |
| **Voter 3:** | No | Yes | Yes |

?

[ Constraint: we have money for *at most two projects* ]

# General Perspective

The last example is actually pretty general. We can rephrase many aggregation problems as problems of *binary aggregation:*

| | |
|---|---|
| *Do you rank option* ○ *above option* ○ *?* | Yes/No |
| *Do you believe formula "$p \rightarrow q$" is true?* | Yes/No |
| *Do you want the new school to get funded?* | Yes/No |

Each problem domain comes with its own *rationality constraints:*

*Rankings should be transitive and not have any cycles.*

*The accepted set of formulas should be logically consistent.*

*We should fund at most two projects.*

The *paradoxes* we have seen show that the *majority rule* does not *lift* our rationality constraints from the *individual* to the *collective* level.

# Binary Aggregation with Integrity Constraints

Basic terminology and notation:

- Set of *individuals* $\mathcal{N} = \{1, \ldots, n\}$; set of *issues* $\mathcal{I} = \{1, \ldots, m\}$.
- Corresponding set of *propositional symbols* $PS = \{p_1, \ldots, p_m\}$ and *propositional language* $\mathcal{L}_{PS}$ interpreted on $\mathcal{D} = \{0, 1\}^m$.
- An *aggregation procedure* is a function $F : \mathcal{D}^{\mathcal{N}} \to \mathcal{D}$. That is, each individual $i \in \mathcal{N}$ votes by submitting a *ballot* $B_i \in \mathcal{D}$.
- An *integrity constraint* is a formula $\mathrm{IC} \in \mathcal{L}_{PS}$ encoding a "rationality assumption". Ballot $B \in \mathcal{D}$ is *rational* iff $B \models \mathrm{IC}$.

Now we can define our main concept:

- An aggregation procedure $F : \mathcal{D}^{\mathcal{N}} \to \mathcal{D}$ is *collectively rational* for $\mathrm{IC} \in \mathcal{L}_{PS}$ if $B_i \models \mathrm{IC}$ for all $i \in \mathcal{N}$ implies $F(B_1, \ldots, B_n) \models \mathrm{IC}$.

# Axioms for Binary Aggregation

Paradoxes show that aggregation is not trivial. We need to carefully formulate what we want, using so-called *axioms*.

- **Unanimity:** For any profile of rational ballots $(B_1, \ldots, B_n)$ and any $x \in \{0, 1\}$, if $b_{i,j} = x$ for all $i \in \mathcal{N}$, then $F(B_1, \ldots, B_n)_j = x$.
- **Anonymity:** For any rational profile $(B_1, \ldots, B_n)$ and any permutation $\pi : \mathcal{N} \to \mathcal{N}$, we get $F(B_1..B_n) = F(B_{\pi(1)}..B_{\pi(n)})$.
- Others: neutrality, independence, monotonicity, ...

Axioms are (usually) defined for a given *domain of aggregation:* those profiles in $\mathcal{D}^{\mathcal{N}}$ that are rational for a given $\mathrm{IC}$.

# Template for Results

Let $\mathcal{L} \subseteq \mathcal{L}_{PS}$ be a *language of integrity constraints*. By fixing $\mathcal{L}$ we fix a range of possible domains of aggregation.

Two ways of defining classes of aggregation procedures:

- The class of procedures defined by a given list of *axioms* AX:

$$\mathcal{F}_{\mathcal{L}}[\text{AX}] \quad := \quad \{F : \mathcal{D}^{\mathcal{N}} \to \mathcal{D} \mid F \text{ satisfies AX on all } \mathcal{L}\text{-domains}\}$$

- The class of procedures that *lift* all integrity constraints in $\mathcal{L}$:

$$\mathcal{CR}[\mathcal{L}] \quad := \quad \{F : \mathcal{D}^{\mathcal{N}} \to \mathcal{D} \mid F \text{ is collect. rat. for all } \mathrm{IC} \in \mathcal{L}\}$$

**What we want:**

$$\mathcal{CR}[\mathcal{L}] \; = \; \mathcal{F}_{\mathcal{L}}[\text{AX}]$$

## Example for a Characterisation Result

*Cubes* (= conjunctions of literals) are lifted by an aggregation procedure *iff* that procedure satisfies *unanimity*:

$$\mathcal{CR}[\text{cubes}] \quad = \quad \mathcal{F}_{\text{cubes}}[\text{Unanimity}]$$

For more results of this sort, see the paper cited below.

U. Grandi and U. Endriss. Lifting Rationality Assumptions in Binary Aggregation. Proc. AAAI-2010.

# Application: Good Binary Aggregation Procedures

Is there a procedure that will lift *every* integrity constraint? *Yes!*

> $F$ will lift *every* $\mathrm{IC} \in \mathcal{L}_{PS}$ *iff* $F$ is a *generalised dictatorship*, i.e., *iff* there exists a function $g : \mathcal{D}^{\mathcal{N}} \to \mathcal{N}$ such that always $F(B_1, \ldots, B_n) = B_{g(B_1,\ldots,B_n)}$.

The class of generalised dictatorships includes:

- proper *dictatorships* $F_i : (B_1, \ldots, B_n) \mapsto B_i$ for fixed $i \in \mathcal{N}$
- *distance-based generalised dictatorships* mapping $(B_1, \ldots, B_n)$ to that $B_i$ that minimises the sum of the Hamming distances to the others (+ tie-breaking). An attractive procedure!

More applications are discussed in the paper cited below.

U. Grandi and U. Endriss. Binary Aggregation with Integrity Constraints. Proc. IJCAI-2011.

# Last Slide

Binary aggregation with integrity constraints:

- *language* to express *rationality assumptions* in binary aggregation
- concept of *collective rationality* with respect to an IC
- characterisation results, relating *axioms* and *languages*
- *applications:* [preference + judgment aggreg.], good procedures

Bigger picture:

- *Axiomatic Method* in SCT: derive sophisticated result for specific domain (with specific rationality assumptions) and specific axioms
- "*AI Approach*": need machinery to reason about many different application-specific domains, rationality assumptions, and axioms

Broader research area:

- Computational Social Choice, see `www.illc.uva.nl/COMSOC/`