Judgment Aggregation with Rationality and Feasibility Constraints

Ulle Endriss
Institute for Logic, Language and Computation
University of Amsterdam

Example

The five members of a local government council have to decide on whether to approve funding for three community initiatives . . .

	School?	**Theatre?**	**Parking?**
Anita	0	0	1
Björn	1	1	1
Christina	1	0	1
Dolph	1	1	0
Zlatan	0	1	1
Majority	1	1	1

Rationality Constraint = "I should support at least one initiative"

Feasibility Constraint = "We cannot afford paying for all initiatives"

Talk Outline

I propose a new model of *judgment aggregation* that distinguishes between *rationality* (input) and *feasibility* (output) constraints. And:

- *Characterisation Theorem* (when does the majority rule "work"?)
- Definition of *Majoritarian Aggregation Rules* (that always "work")
- Application: *Simulating Common Voting Rules*

This talk is based on the paper cited below.

U. Endriss. JA with Rationality and Feasibility Constraints. AAMAS-2018.

The Model

The *agenda* is a finite set of propositions you may *accept* or *reject*.

A *judgment* is a function $J : \mathrm{Agenda} \to \{0, 1\}$.

We ask n *agents* to each provide us with a judgment (today: n is odd).

An *aggregation rule* F maps any given *profile* $\boldsymbol{J} = (J_1, \ldots, J_n)$ of judgments, one for each agent, to a single compromise judgment.

Can describe *rationality* (input) and *feasibility* (output) *constraints* using propositional logic. For $\mathrm{Agenda} = \{S, T, P\}$ we might use:

$$\textsc{Rat} = S \vee T \vee P \qquad \textsc{Feas} = \neg(S \wedge T \wedge P)$$

What we would like:

$$(J_1, \ldots, J_n) \in \mathrm{Mod}(\textsc{Rat})^n \quad \Longrightarrow \quad F(J_1, \ldots, J_n) \in \mathrm{Mod}(\textsc{Feas})$$

Rationality and Feasibility

Standard JA does not distinguish between *rationality* and *feasibility*. Rather, *all* judgments are supposed to be "consistent". *Why?*

- Examples used in early papers do not require such a distinction ("contract breach", "hiring committee", . . .).
- The basic *discursive dilemma* and technical work arising from it work beautifully with a single constraint (such as $R \leftrightarrow P \wedge Q$).

P	Q	R
1	1	1
1	0	0
0	1	0
1	1	0

But (I think) most interesting applications require this distinction:

- *participatory budgeting* (individual rationality $\neq$ budget feasibility)
- *decision making* (compromise may relax original specifications)
- *voting* (individual preference rankings $\neq$ election outcomes)

Characterisation Theorem for the Majority Rule

When can we use the majority rule without risking infeasible outcomes?

Need some terminology:

- A formula is *simple* if it is equivalent to a conjunction of 2-clauses.
- The *prime implicates* of a formula are the logically strongest clauses that are entailed by that formula.

Theorem: *The majority rule guarantees feasible outcomes on all rational profiles iff these two conditions are satisfied:*

- *The feasibility constraint is entailed by the rationality constraint.*
- *Every nonsimple prime implicate of the feasibility constraint is entailed by a simple prime implicate of the rationality constraint.*

This generalises a seminal result by Nehring and Puppe (2007).

K. Nehring and C. Puppe. The Structure of Strategy-proof Social Choice. Part I: General Characterization and Possibility Results on Median Space. *JET*, 2007.

Proof Sketch: Characterisation Theorem

Fact: *If* $\Gamma \models \Gamma'$ *and* π' *is a PI of* Γ'*, then* $\pi \models \pi'$ *for some PI* π *of* Γ*.*

Lemma: If a *unanimous* F guarantees feasibility, then $\textsc{Rat} \models \textsc{Feas}$.
Proof: consider $\boldsymbol{J} = (J, \ldots, J)$ with $J \in \mathrm{Mod}(\textsc{Rat}) \setminus \mathrm{Mod}(\textsc{Feas})$ ...

So restrict attention to cases with $\textsc{Rat} \models \textsc{Feas}$ when proving theorem:

($\Rightarrow$) Suppose simplicity condition fails: $\textsc{Feas}$ has nonsimple PI π' of form $\varphi_1 \vee \varphi_2 \vee \varphi_3 \vee \psi'$ s.t. $\pi \models \pi'$ for no simple PI π of $\textsc{Rat}$.
But $\textsc{Rat}$ must have *some* (thus nonsimple) PI π s.t. $\pi \models \pi'$.
So π must be of form $\varphi_1 \vee \varphi_2 \vee \varphi_3 \vee \psi$ with $\psi \models \psi'$.
Build (rational) profile where $\lfloor \frac{n}{3} \rfloor$ accept φ_1, $\lceil \frac{n}{3} \rceil$ accept φ_2, rest accept φ_3, all reject ψ' (and ψ). Then majority reject $\textsc{Feas}$. ✓

($\Leftarrow$) Suppose simplicity condition holds. Then $\textsc{Rat}$ must have PI of form $\varphi_1 \vee \varphi_2$ that entails $\textsc{Feas}$. So all must accept $\varphi_1 \vee \varphi_2$.
W.l.o.g. majority accept φ_1. So majority accept $\textsc{Feas}$. ✓.

Majoritarian Aggregation Rules

The majority rule might return infeasible outcomes. So we need rules that "approximate" the ideal of the majority *and* guarantee feasibility:

$$\text{max-set}(\boldsymbol{J}, \textsc{Feas}) = \underset{J \in \text{Mod}(\textsc{Feas})}{\text{argsetmax}} \{\varphi \in \text{Agenda} : J(\varphi) = \text{Maj}(\boldsymbol{J})(\varphi)\}$$

$$\text{max-num}(\boldsymbol{J}, \textsc{Feas}) = \underset{J \in \text{Mod}(\textsc{Feas})}{\text{argmax}} |\{\varphi \in \text{Agenda} : J(\varphi) = \text{Maj}(\boldsymbol{J})(\varphi)\}|$$

$$\text{max-sum}(\boldsymbol{J}, \textsc{Feas}) = \underset{J \in \text{Mod}(\textsc{Feas})}{\text{argmax}} \sum_{i \in \text{Agents}} |\{\varphi \in \text{Agenda} : J(\varphi) = J_i(\varphi)\}|$$

Other options: lexi-max, greedy-max, . . .

Remark: All return only the *majority judgment* $\text{Maj}(\boldsymbol{J})$ when feasible.

Remark: These rules are known under (various, often confusing) other names for the standard model of JA (with $\textsc{Rat} = \textsc{Feas}$).

Preference Aggregation as Judgment Aggregation

Main topic in SCT is *preference aggregation*. Can encode this in JA using the agenda $\{p_{x \succcurlyeq y} \mid x, y \in \text{Alternatives}\}$ and this constraint:

$$\bigwedge_{x,y,z} (p_{x \succcurlyeq y} \wedge p_{y \succcurlyeq z} \rightarrow p_{x \succcurlyeq z}) \;\wedge\; \bigwedge_{x,y} (p_{x \succcurlyeq y} \vee p_{y \succcurlyeq x}) \;\wedge\; \bigwedge_{x \neq y} \neg (p_{x \succcurlyeq y} \wedge p_{y \succcurlyeq x})$$

Very useful! For example, can simulate classical Condorcet paradox:

	$p_{x \succcurlyeq y}$	$p_{y \succcurlyeq z}$	$p_{x \succcurlyeq z}$	$\cdots$	*corresponding order*
Agent 1	1	1	1		$x \succ y \succ z$
Agent 2	0	1	0		$y \succ z \succ x$
Agent 3	1	0	0		$z \succ x \succ y$
Majority	1	1	0		*not a total order*

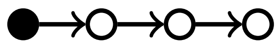
Also useful to get deeper understanding of Arrow's impossibility and to obtain complexity results in JA. *Great!* But: how simulate *voting rules*?

Simulating Common Voting Rules

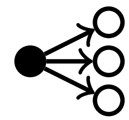
While embedding preference aggregation is a basic staple of the JA literature, for many voting rules it has been difficult to simulate them.

Refining an idea by Lang and Slavkovik (2013), we can do better.

Can express relevant *constraints:*

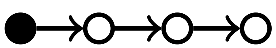
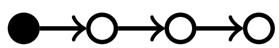
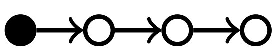
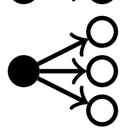
 =

$$\bigwedge_{x,y,z} (p_{x\succcurlyeq y} \wedge p_{y\succcurlyeq z} \rightarrow p_{x\succcurlyeq z}) \wedge \cdots$$

 =

$$\bigvee_{x} \bigwedge_{y\neq x} (p_{x\succcurlyeq y} \wedge \neg p_{y\succcurlyeq x}) \wedge \cdots$$

This yields the following *simulation results:*

Rationality	**Feasibility**	**max-set**	**max-num**	**max-sum**
		Top Cycle	Slater	Kemeny
		Uncovered Set	Copeland	Borda

J. Lang and M. Slavkovik. Judgment Aggreg. Rules and Voting Rules. ADT-2013.

Proof Sketch: Simulating Borda

Borda's rule for m alternatives: every voter awards $m-1$ points to the alternative she ranks first, $m-2$ to the one she ranks second, etc.

Claim: max-sum simulates *Borda* when the rationality constraint forces *total orders* and the feasibility constraint forces "*stars*".

Recall:

$$\text{max-sum}(\boldsymbol{J}, \textsc{Feas}) = \operatorname*{argmax}_{J \in \text{Mod}(\textsc{Feas})} \sum_{i \in \text{Agents}} |\{\varphi \in \text{Agenda} : J(\varphi) = J_i(\varphi)\}|$$

Idea: Returning a star means choosing a winning alternative $x^\star$.
Agreement = number of accepted $p_{x^\star \succcurlyeq x}$ in profile (for any $x \neq x^\star$).
But that's Borda (summing by alternative rather than by agent)! ✓

Last Slide

What just happened:

- New model of JA that emphasises *rationality* and *feasibility*
- Feasibility of *majority rule:* characterisation via *prime implicates*
- Feasible *aggregation rules:* max-set, max-num, max-sum
- Convincing *embedding* of *Borda* voting rule (and others) into JA