Rationalisation of Profiles of Abstract Argumentation Frameworks

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Motivation

Central question in MAS research is how to aggregate diverse “views” of several agents. Also relevant: what diversity is actually possible?

We consider this second, less commonly asked question:

- we model “views” as abstract argumentation frameworks
- individual view is mix of “facts” and “preferences”
- can we rationalise diverse observations by disentangling them?

Talk Outline

• Background: value-based variant of abstract argumentation
• Concept: formal definition of the rationalisability problem
• Results: single-agent case and multiagent case
Value-Based Argumentation

An argumentation framework \( AF = \langle \text{Arg}, \rightarrow \rangle \) consists of a finite set of arguments \( \text{Arg} \) and a binary attack-relation \( \rightarrow \).

An audience-specific value-based AF \( \langle \text{Arg}, \rightarrow, \text{Val}, \text{val}, \geq \rangle \) consists of an AF \( \langle \text{Arg}, \rightarrow \rangle \), a labelling \( \text{val} : \text{Arg} \rightarrow \text{Val} \) of arguments with values, and a (reflexive and transitive) preference order \( \geq \) on \( \text{Val} \).

Argument \( A \) defeats \( B \) (\( A \Rightarrow B \)) if \( A \rightarrow B \) but \( \text{val}(B) \geq \text{val}(A) \).

Note that \( \langle \text{Arg}, \rightarrow \rangle \) is itself just another AF.

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The Rationalisability Problem

Given $n$ agents and a profile of AF’s $(\langle Arg_1, \Rightarrow_1 \rangle, \ldots, \langle Arg_n, \Rightarrow_n \rangle)$ the rationalisability problem asks whether there exist:

- a master attack-relation $\rightarrow$ on $Arg = Arg_1 \cup \cdots \cup Arg_n$
- a set of values $Val$ and a value-labelling $val : Arg \rightarrow Val$
- a profile of preference orders $(\succeq_1, \ldots, \succeq_n)$

such that $A \Rightarrow_i B$ iff $A \rightarrow B$ but $val(B) \succ_i val(A)$ [for all $i, A, B$].

We may also wish to impose certain constraints on allowed solutions.
The Single-Agent Case: Example

Let $\text{Arg} = \{A, B, C\}$. Suppose the master attack-relation $\rightarrow$ is fixed.

observed defeat-relation $\Rightarrow$ fixed master attack-relation $\rightarrow$

Can you rationalise $\Rightarrow$ in terms of $\rightarrow$ using . . .

- up to two values?
- up to three values?
- up to three values and a complete preference order?
The Single-Agent Case: Results

Can you rationalise a given AF $\langle \text{Arg}, \Rightarrow \rangle$ by means of some master attack-relation $\rightarrow$, value-labelling $\text{val} : \text{Arg} \rightarrow \text{Val}$, and preference $\geq$?

Depends on the constraints:

- **No constraints** (or only on value-labelling): always yes!

  *Just let $(\rightarrow) = (\Rightarrow)$, use whatever value-labelling, and let $\geq$ be indifferent between any two arguments.*

- **Fixed master attack-relation**: easy *polynomial* algorithm

  *Assign unique value to each argument. Just need to check $(\Rightarrow) \subseteq (\rightarrow)$, removed part $(\rightarrow \setminus \Rightarrow)$ is acyclic, and preference does not cancel too many attacks: $(\Rightarrow) \cap (\rightarrow \setminus \Rightarrow)^+ = \emptyset$.*

- **Bound on values and complete preference**: also polynomial

  *Encode as integer program with 2 variables per inequality.*

For (possibly) *incomplete* preferences this is an *open problem*. 
Rationalisation with Bound on Number of Values

Can you rationalise $\langle \text{Arg}, \Rightarrow \rangle$ by means of master attack-relation $\rightarrow$, some $val: \text{Arg} \rightarrow \text{Val}$ with $|\text{Val}| \leq k$, and some complete $\geq$?

Suppose master attack-relation $\rightarrow$ is given [if not: $(\rightarrow) = (\Rightarrow)$ is best].
W.l.o.g., assume $(\Rightarrow) \subseteq (\rightarrow)$ [otherwise: not rationalisable].
W.l.o.g., let $\text{Val} = \{1, \ldots, k\}$ and let $\geq$ be $\geq$ on the natural numbers.

For every $A \in \text{Arg}$, introduce integer variable $x_A$ with $1 \leq x_A \leq k$.

Construct an integer program with these inequalities:

- $x_A < x_B$ whenever $A \rightarrow B$ but not $A \Rightarrow B$
- $x_B \leq x_A$ whenever $A \Rightarrow B$ [and thus also $A \rightarrow B$]

Deciding feasibility of IP’s with 2 variables per inequality is polynomial!

Crucial: modelling $val(B) \succ val(A)$ as $x_B \leq x_A$ rather than $x_B \succ x_A$ is ok only due to the completeness requirement!
Multiagent Rationalisation: Example

Let \( \text{Arg} = \{A, B, C\} \) and \( n = 2 \). Try to rationalise the following profile.

- First defeat-relation \( \models_1 \)
- Second defeat-relation \( \models_2 \)

A few hints:

- Rationalisable if rationalisable with \( (\rightarrow) = (\models_1) \cup (\models_2) = (\models_1) \).
- Rationalisable if rationalisable with one value for each argument.
- Now try to build \( \succeq_2 \) (preference order of second agent) \ldots
Multiagent Rationalisation: Easy Cases

In our example, it was *impossible to decompose* the problem and to consider rationalisability separately for each agent.

But when all constraints are of these types, then you *can decompose*:

- the master attack-relation $\rightarrow$ is fixed
- the value-labelling $\text{val} : (\text{Arg}_1 \cup \cdots \cup \text{Arg}_n) \rightarrow \text{Val}$ is fixed

So multiagent rationalisability *reduces* to single-agent rationalisability! Thus, multiagent rationalisability is *polynomial* in these cases:

- no constraints
- only the master attack-relation is fixed
- only the value-labelling is fixed
- master attack-relation and value-labelling are fixed

*Single-agent rationalisability is also easy [case not discussed before].
Multiagent Rationalisation: Hard (and Easy) Cases

Bad news: Let $k \geq 3$. For constraint $|Val| \leq k$, rationalisability is \textit{NP-complete} (whether or not the master attack-relation is given).

- Proof by reduction from \textsc{Graph Colouring}.
- Open problem whether also \textit{NP-complete} for $Arg_1 = \cdots = Arg_n$.

Good news: for $k = 2$ there is a \textit{polynomial} algorithm [not in paper].

Good news: for “large” \textit{bounds} it’s also \textit{polynomial}: $k \in \Omega(|Arg|)$. 

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Last Slide

We have introduced the *rationalisability problem* for a given profile of argumentation frameworks, one for each agent in a multiagent system:

- identified various cases that admit *polynomial algorithms*
- but multiagent case with bound on values is *NP-complete*
- several *open problems* regarding complexity

Definition of the rationalisability problem in terms of Bench-Capon’s *value-based* argumentation frameworks, but basic idea is general.

Possible *application* scenarios:

- to determine relevant profiles for research on aggregating AF’s
- if rationalisable, we can use preference aggregation instead
- to spot inconsistencies in online debating platforms