Fairness and Efficiency in Multiagent Resource Allocation

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Computational Social Choice

- Research at the interface of
  - mathematical economics: social choice, game theory, decision theory
  - computer science and AI, multiagent systems, logic

- Some examples:
  - voting: computational hardness as a barrier against manipulation
  - preference representation in combinatorial domains
  - logic-based modelling of social choice procedures
  - multiagent resource allocation and fair division

Talk Overview

• Introduction to Multiagent Resource Allocation

• Preliminaries on Distributed Negotiation Framework

• A couple of examples for so-called convergence results. Issues:
  – Under what circumstances can we hope that a system where agents negotiate autonomously and locally will converge to a state considered optimal from a global point of view?
  – How should we actually define such a notion of global optimality? “fairness and efficiency”
Multiagent Resource Allocation (MARA)

A tentative definition would be the following:

MARA is the process of distributing a number of items amongst a number of interested parties.

What items? This talk is about the allocation of indivisible goods.

Some questions to think about:

- How are these items being distributed (allocation procedure)?
- Why are they being distributed? What’s a “good” allocation?

Choice of Allocation Procedure

An allocation procedure to determine a suitable allocation of resources may be either centralised or distributed:

- In the *centralised* case, a single entity decides on the final allocation, possibly after having elicited the preferences of the other agents. Example: combinatorial auctions

- In the *distributed* case, allocations emerge as the result of a sequence of local negotiation steps. Such local steps may or may not be subject to structural restrictions (say, bilateral deals).
Distributed Negotiation Framework

• Set of agents $A = \{1..n\}$ and finite set of indivisible resources $R$.

• An allocation $A$ is a partitioning of $R$ amongst the agents in $A$.
  
  Example: $A(i) = \{r_5, r_7\}$ — agent $i$ owns resources $r_5$ and $r_7$

• Each agent $i \in A$ has got a valuation function $v_i : 2^R \rightarrow \mathbb{R}$.
  
  Example: $v_i(A) = v_i(A(i)) = 577.8$ — agent $i$ is pretty happy

• Agents may engage in negotiation to exchange resources in order to benefit either themselves or society as a whole.

• A deal $\delta = (A, A')$ is a pair of allocations (before/after).

• A deal may come with a number of side payments to compensate some of the agents for a loss in valuation. A payment function is a function $p : A \rightarrow \mathbb{R}$ with $\sum_{i \in A} p(i) = 0$.

  Example: $p(i) = 5$ and $p(j) = -5$ means that agent $i$ pays $5$, while agent $j$ receives $5$. 
The Local/Individual Perspective

A rational agent (who does not plan ahead) will only accept deals that improve its individual welfare:

**Definition 1 (IR)** A deal $\delta = (A, A')$ is called *individually rational* iff there exists a payment function $p$ such that $v_i(A') - v_i(A) > p(i)$ for all $i \in A$, except possibly $p(i) = 0$ for agents $i$ with $A(i) = A'(i)$.

That is, an agent will only accept a deal if it results in a gain in value (or money) that strictly outweighs a possible loss in money (or value).

The Global/Social Perspective

**Definition 2 (Social welfare)** The (utilitarian) *social welfare* of an allocation of resources $A$ is defined as follows:

$$sw(A) = \sum_{i \in \text{Agents}} v_i(A)$$
Example

Let $\mathcal{A} = \{ann, bob\}$ and $\mathcal{R} = \{chair, table\}$ and suppose our agents use the following valuation functions:

\[
\begin{align*}
    v_{ann}(\emptyset) &= 0 & v_{bob}(\emptyset) &= 0 \\
    v_{ann}(\{chair\}) &= 2 & v_{bob}(\{chair\}) &= 3 \\
    v_{ann}(\{table\}) &= 3 & v_{bob}(\{table\}) &= 3 \\
    v_{ann}(\{chair, table\}) &= 7 & v_{bob}(\{chair, table\}) &= 8
\end{align*}
\]

Furthermore, suppose the initial allocation of goods is $A_0$ with $A_0(ann) = \{chair, table\}$ and $A_0(bob) = \{\}$. Social welfare for allocation $A_0$ is 7, but it could be 8. By moving only a *single* good from agent $ann$ to agent $bob$, the former would lose more than the latter would gain (not IR).

The only possible deal would be to move the whole set $\{chair, table\}$. 
Negotiating Socially Optimal Allocations

So, under what circumstances can we hope that agents may be able to negotiate a socially optimal allocation?

If we do not impose any structural restrictions on deals, then we can get a strong convergence result:

**Theorem 1 (Sandholm, 1998)** Any sequence of IR deals will eventually result in an allocation with maximal social welfare.

How can we explain this positive result? ...
Linking the Local and the Global Perspective

IR deals turn out to be exactly those deals that increase social welfare:

Lemma 2 (Individual rationality and social welfare) A deal \( \delta = (A, A') \) is IR iff \( sw(A) < sw(A') \).

Proof: "\( \Rightarrow \)" : IR means that overall valuation gains outweigh overall payments (which are \( = 0 \)). "\( \Leftarrow \)" : Using side payments, the social surplus can be divided amongst all deal participants. ✓

Convergence then follows from the fact that the overall space of allocations is finite.

Remark: Lemma 2 also suggests that our notion of rationality is somehow "appropriate" for agents living in a utilitarian society . . .

Efficiency and Fairness

When assessing the quality of an allocation we can distinguish (at least) two types of indicators of social welfare.

Aspects of **efficiency** (*not* in the computational sense) include:

- The sum of payoffs should be as high as possible (**utilitarianism**).
- The chosen agreement should be such that there is no alternative allocation that would be better for some and not worse for any of the other agents (**Pareto optimality**).

Aspects of **fairness** include:

- The agent that is going to be worst off should be as well off as possible (**egalitarianism**).
- No agent should prefer to take the bundle allocated to one of its peers rather than keeping their own (**envy-freeness**).
Research Agenda

Theorem 1 has been an example (the first) for a convergence result. We would like to better understand under what circumstances convergence is possible for different scenarios. Parameters:

- The class of *valuation functions* considered:
  - arbitrary or, say, only additive valuations

- The type of *structural restrictions* imposed on deals:
  - general multilateral deals, bilateral deals, 1-resource deals, . . .

- The *rationality criterion* used to define individual agent behaviour:
  - individual rationality or something else

- The *social welfare criterion* used to define the global goal:
  - utilitarian or egalitarian social welfare, envy-freeness, . . .

It’s particularly hard to achieve envy-freeness, because a local deal in one part of society can make another agent somewhere else envious . . .
Envy-free States

Unfortunately, there are cases where envy-free allocations do not exist. Example: 2 agents, 1 good desired by both

We can try to circumvent this problem by taking the balance of past side payments into account when defining envy-freeness:

- Associate each allocation $A$ with a payment balance $\pi : A \rightarrow \mathbb{R}$, mapping agents to the sum of payments they have made so far.

- A state $(A, \pi)$ is a pair of an allocation and a payment balance.

- Each agent $i \in A$ has got a (quasi-linear) utility function $u_i : 2^\mathcal{R} \times \mathbb{R} \rightarrow \mathbb{R}$, defined as follows: $u_i(R, x) = v_i(R) - x$.

- A state $(A, \pi)$ is envy-free iff $u_i(A(i), \pi(i)) \geq u_i(A(j), \pi(j))$ for all agents $i, j \in A$. An efficient envy-free (EEF) state is an envy-free state maximising utilitarian social welfare.
Envy-freeness and Individual Rationality

By a known result from social choice theory, EEF states always exist (in the presence of money). But we want to find them by means of rational negotiation. Unfortunately, this is generally impossible.

Example: 2 agents, 1 item $r$ with $v_1(\{r\}) = 4$ and $v_2(\{r\}) = 7$. Agent 1 owns $r$ to begin with; giving it to agent 2 would be efficient.

- An IR deal would require a payment within interval $(4, 7)$.
- But to ensure envy-freeness, the payment should be in $[2, 3.5]$.

Compromise: We shall enforce an initial equitability payment

$$\pi_0(i) = v_i(A_0) - sw(A_0)/n$$

before beginning negotiation.
Globally Uniform Payments

Because of the “non-local effects of local deals” in view of envy-freeness, to have any chance of getting a convergence result for EEF states, we will have to restrict the freedom of agents a little by fixing a specific payment function (still IR!):

- Let $\delta = (A, A')$ be an IR deal. The payments as given by the globally uniform payment function (GUPF) are defined as follows:

\[
p(i) = [v_i(A') - v_i(A)] - [sw(A') - sw(A)]/n.
\]

That is, we distribute the (positive!) social surplus to all agents.
Convergence to EEF States

A final restriction concerns agent valuations. *Supermodular* valuations are valuations satisfying the following condition for all $R_1, R_2 \subseteq \mathcal{R}$:

$$v(R_1 \cup R_2) \geq v(R_1) + v(R_2) - v(R_1 \cap R_2)$$

We are now ready to state the result:

**Theorem 3 (Convergence)** If all valuations are supermodular and if initial equitability payments have been made, then any sequence of IR deals using the GUPF will eventually result in an EEF state.

**Generalisation:** if agents are nodes on a graph and can only envy and negotiate with agents they are linked to, then we get a similar result.


Conclusions

• Examples of recent work in Multiagent Resource Allocation

• Technical results about convergence to socially optimal states

• Two issues that are special about this line of work:
  – distributed resource allocation via multilateral deals
  – consideration of fairness criteria, not just efficiency

• Many open questions and topics for future work . . .

• Papers are available from my website:
  
  http://www.illc.uva.nl/~ulle/