Binary Aggregation with Integrity Constraints

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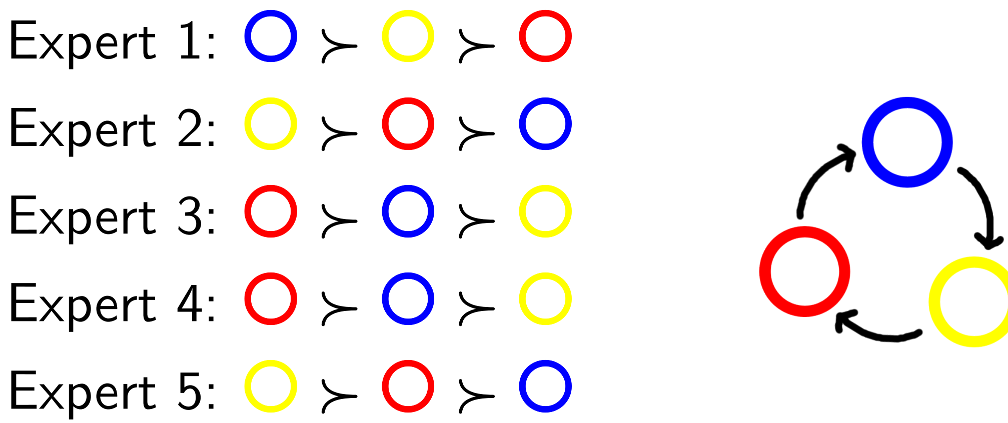
Social Choice and the Condorcet Paradox

Social Choice Theory asks: how should we aggregate the preferences of the members of a group to obtain a "social preference"?



Marie Jean Antoine Nicolas de Caritat (1743–1794), better known as the **Marquis de Condorcet**: Highly influential Mathematician, Philosopher, Political Scientist, Political Activist. Observed that the *majority rule* may produce inconsistent outcomes ("Condorcet Paradox").

A Classic: Arrow's Impossibility Theorem

In 1951, K.J. Arrow published his famous *Impossibility Theorem:*

Any preference aggregation mechanism for *three* or more alternatives that satisfies the axioms of *unanimity* and *IIA* must be *dictatorial*.

- Unanimity: if everyone says $A \succ B$, then so should society.
- Independence of Irrelevant Alternatives (IIA): if society says $A \succ B$ and someone changes their ranking of C, then society should still say $A \succ B$.

Kenneth J. Arrow (born 1921): American Economist; Professor Emeritus of Economics at Stanford; Nobel Prize in Economics 1972 (youngest recipient ever). His 1951 PhD thesis started modern Social Choice Theory. Google Scholar lists 9557 citations of the thesis.

Social Choice and AI (1)

Social choice theory has natural applications in AI:

- *Search Engines:* to determine the most important sites based on links ("votes") + to aggregate the output of several search engines
- *Recommender Systems:* to recommend a product to a user based on earlier ratings by other users
- *Multiagent Systems:* to aggregate the beliefs + to coordinate the actions of groups of autonomous software agents
- *AI Competitions:* to determine who has developed *the best* trading agent / SAT solver / RoboCup team

But not all of the classical assumptions will fit these new applications. So AI needs to develop *new models* and *ask new questions*.

Social Choice and AI (2)

Vice versa, techniques from AI, and computational techniques in general, are useful for advancing the state of the art in social choice:

- *Algorithms and Complexity:* to develop algorithms for (complex) voting procedures + to understand the hardness of "using" them
- *Knowledge Representation:* to compactly represent the preferences of individual agents over large spaces of alternatives
- *Logic and Automated Reasoning:* to formally model problems in social choice + to automatically verify (or discover) theorems

Indeed, you will find many papers on social choice at AI conferences (e.g., IJCAI, ECAI, AAAI, AAMAS) and many AI researchers participate in events dedicated to social choice (e.g., COMSOC).

Y. Chevaleyre, U. Endriss, J. Lang, and N. Maudet. Preference Handling in Combinatorial Domains: From AI to Social Choice. *AI Magazine*, 29(4):37–46, 2008.

Rest of this Talk

- Some more examples for paradoxes of aggregation
- General framework: *binary aggregation with integrity constraints*
- New idea: lifting rationality assumptions
- Applications of that idea

Judgment Aggregation

	p	$p \to q$	q
Judge 1:	True	True	True
Judge 2:	True	False	False
Judge 3:	False	True	False
		?	

Multiple Referenda

	fund museum?	*fund school?*	*fund metro?*
Voter 1:	Yes	Yes	No
Voter 2:	Yes	No	Yes
Voter 3:	No	Yes	Yes

?

[Constraint: we have money for *at most two projects*]

General Perspective

The last example is actually pretty general. We can rephrase many aggregation problems as problems of *binary aggregation:*

Do you rank option ○ *above option* ○ *?* Yes/No

Do you believe formula "$p \to q$" is true? Yes/No

Do you want the new school to get funded? Yes/No

Each problem domain comes with its own *rationality constraints:*

Rankings should be transitive and not have any cycles.

The accepted set of formulas should be logically consistent.

We should fund at most two projects.

The *paradoxes* we have seen show that the *majority rule* does not *lift* our rationality constraints from the *individual* to the *collective* level.

Binary Aggregation with Integrity Constraints

Basic terminology and notation:

- Set of *individuals* $\mathcal{N} = \{1, \ldots, n\}$; set of *issues* $\mathcal{I} = \{1, \ldots, m\}$.
- Corresponding set of *propositional symbols* $PS = \{p_1, \ldots, p_m\}$ and *propositional language* $\mathcal{L}_{PS}$ interpreted on $\mathcal{D} = \{0, 1\}^m$.
- An *aggregation procedure* is a function $F : \mathcal{D}^{\mathcal{N}} \to \mathcal{D}$. That is, each individual $i \in \mathcal{N}$ votes by submitting a *ballot* $B_i \in \mathcal{D}$.
- An *integrity constraint* is a formula $\mathrm{IC} \in \mathcal{L}_{PS}$ encoding a "rationality assumption". Ballot $B \in \mathcal{D}$ is *rational* iff $B \models \mathrm{IC}$.

Now we can define our main concept:

- An aggregation procedure $F : \mathcal{D}^{\mathcal{N}} \to \mathcal{D}$ is *collectively rational* for $\mathrm{IC} \in \mathcal{L}_{PS}$ if $B_i \models \mathrm{IC}$ for all $i \in \mathcal{N}$ implies $F(B_1, \ldots, B_n) \models \mathrm{IC}$.

Axioms for Binary Aggregation

Paradoxes show that aggregation is not trivial. We need to carefully formulate what we want: "*axioms*".

- **Unanimity:** For any profile of rational ballots $(B_1, \dots, B_n)$ and any $x \in \{0, 1\}$, if $b_{i,j} = x$ for all $i \in \mathcal{N}$, then $F(B_1, \dots, B_n)_j = x$.
- **Anonymity:** For any rational profile $(B_1, \dots, B_n)$ and any permutation $\sigma : \mathcal{N} \to \mathcal{N}$, we get $F(B_1..B_n) = F(B_{\sigma(1)}..B_{\sigma(n)})$.
- Others: neutrality, independence, monotonicity, ...

Axioms are (usually) defined for a given *domain of aggregation:* those profiles in $\mathcal{D}^{\mathcal{N}}$ that are rational for a given IC.

Template for Results

Let $\mathcal{L} \subseteq \mathcal{L}_{PS}$ be a *language of integrity constraints*. By fixing $\mathcal{L}$ we fix a range of possible domains of aggregation.

Two ways of defining classes of aggregation procedures:

- The class of procedures defined by a given list of *axioms* AX:

$$\mathcal{F}_{\mathcal{L}}[\text{AX}] \quad := \quad \{F : \mathcal{D}^{\mathcal{N}} \rightarrow \mathcal{D} \mid F \text{ satisfies AX on all } \mathcal{L}\text{-domains}\}$$

- The class of procedures that *lift* all integrity constraints in $\mathcal{L}$:

$$\mathcal{CR}[\mathcal{L}] \quad := \quad \{F : \mathcal{D}^{\mathcal{N}} \rightarrow \mathcal{D} \mid F \text{ is collect. rat. for all } \mathrm{IC} \in \mathcal{L}\}$$

What we want:

$$\mathcal{CR}[\mathcal{L}] \; = \; \mathcal{F}_{\mathcal{L}}[\text{AX}]$$

Example for a Characterisation Result

Cubes (= conjunctions of literals) are lifted by an aggregation procedure *iff* that procedure satisfies *unanimity*:

$$\mathcal{CR}[\text{cubes}] \quad = \quad \mathcal{F}_{\text{cubes}}[\text{Unanimity}]$$

More Results

Characterisation results (selection):

- F lifts all constraints $p_j \leftrightarrow p_k$ *iff* F is *issue-neutral*
- F lifts all constraints $p_j \leftrightarrow \neg p_k$ *iff* F is *domain-neutral*

Negative results:

- there exists *no language* that characterises *anonymous* procedures
- there exists *no language* that characterises *independent* procedures

Application: Good Binary Aggregation Procedures

Is there a procedure that will lift *every* integrity constraint? *Yes!*

> F will lift *every* $\mathrm{IC} \in \mathcal{L}_{PS}$ *iff* F is a *generalised dictatorship*, i.e., *iff* there exists a function $g : \mathcal{D}^{\mathcal{N}} \to \mathcal{N}$ such that always $F(B_1, \ldots, B_n) = B_{g(B_1, \ldots, B_n)}$.

The class of generalised dictatorships includes:

- proper *dictatorship* $F_i : (B_1, \ldots, B_n) \mapsto B_i$ for each $i \in \mathcal{N}$
- *distance-based generalised dictatorships* mapping $(B_1, \ldots, B_n)$ to that B_i minimising the sum of the Hamming distances to the others (+ tie-breaking). An attractive procedure!

Application: New Result in Preference Aggregation

We can translate Arrovian preference aggregation (for linear orders) into binary aggregation with integrity constraints:

- Introduce propositional symbols p_{xy} to mean "x is better than y".
- Include integrity constraints for irreflexivity ($\neg p_{xx}$), completeness ($p_{xy} \vee p_{yx}$), and transitivity ($p_{xy} \wedge p_{yz} \rightarrow p_{xz}$).

Call a preference aggregator *imposed* if there exist x and y such that x is collectively preferred to y in every profile. New theorem:

> *Any anonymous, independent and monotonic aggregator for at least three alternatives and at least two individuals is imposed.*

This is similar to (but different from) Arrow's Theorem.

The proof technique is new: use a "lifting theorem" to narrow down the class of procedures that are CR for above ICs.

Last Slide

Binary aggregation with integrity constraints:

- *language* to express *rationality assumptions* in binary aggregation
- concept of *collective rationality* with respect to an IC
- characterisation results, relating *axioms* and *languages*
- *applications:* preference + judgment aggregation, good procedures

Bigger picture:

- *Axiomatic Method* in SCT: derive sophisticated result for specific domain (with specific rationality assumptions) and specific axioms
- "*AI Approach*": need machinery to reason about many different application-specific domains, rationality assumptions, and axioms

Broader research area:

- Computational Social Choice, see `www.illc.uva.nl/COMSOC/`

U. Grandi and U. Endriss. Lifting Rationality Assumptions in Binary Aggregation. Proc. AAAI-2010.