Distribution Rules Under Dichotomous Preferences

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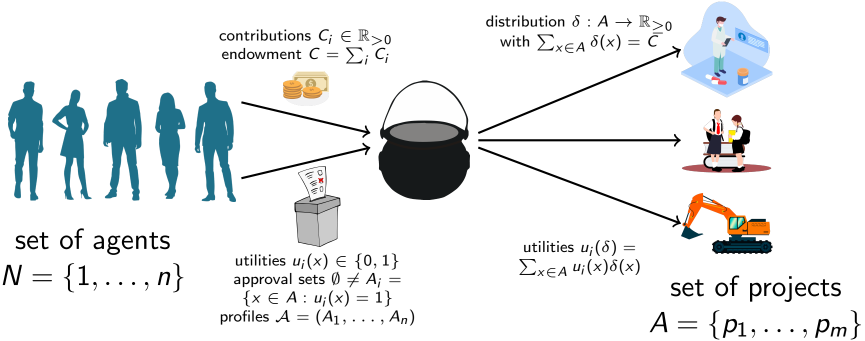
Abstract

- Framework for distribution of divisible resource
- Axiomatic analysis of 4 distribution rules, one is newly introduced
- Impossibility result: No strategyproof, efficient rule can guarantee that at least one approved project per agent receives positive amount of resource

The framework



A distribution rule f assigns to every profile $\mathcal{A}$ a distribution $f(\mathcal{A})$.

The impossibility result

No distribution rule satisfies efficiency, strategyproofness, and positive share when $m \geq 4$ and $n \geq 6$.

Efficiency: A distribution dominates another one if one agent has a strictly higher utility and no agent has a strictly lower utility w.r.t. that distribution. Distribution rule f is efficient if none of its outputs $f(\mathcal{A})$ is dominated by some distribution.

Strategyproofness: No agent can receive a strictly higher utility by lying, i. e. $\forall i, \mathcal{A}, A'_i : u_i(f(\mathcal{A})) \geq u_i(f(\mathcal{A}_{-i}, A'_i))$.

Positive share: No agent is ignored by the rule, i. e. at least one project that they approve of receives funds, $\forall i : u_i(\delta) > 0$.

How to encode the problem?

Linear Programming? ✗

Instead, use *SAT solving* by introducing binary variables $p_{\mathcal{A},M}$ which evaluate to true iff $M \in \mathcal{P}(A) \setminus \{\emptyset\}$ is the support of the distribution $f(\mathcal{A})$

Can we express the axioms in terms of the support?

- Positive share ✓
- Efficiency ✓ (needs a bit of work)
- Strategyproofness ✗

Pessimistic strategyproofness ✓: An agent does not have an incentive to lie in order to obtain optimal utility C, i. e.

$$\forall i, \mathcal{A}, A_i' : u_i(f(\mathcal{A}_{-i}, A_i')) = C \rightarrow u_i(f(\mathcal{A})) = C$$

How to reduce the size?

Is this feasible? ✗

For $m = 4$, $n = 6$, there are $15^6 \approx 11$ Million profiles and 15 different supports, yielding approximately 170 Million variables $p_{\mathcal{A},M}$

Using anonymity and neutrality, we can reduce this down to only 33.000 variables. Easy!

Idea: Drop neutrality and anonymity one by one, i. e.

1. SAT-solve CNF expressing anonymity + efficiency (E) + pessimistic strategyproofness (PSP) + positive share (PS) ($\approx$ 77.000 variables)
2. Extract MUS (only referencing 81 profiles)
3. SAT-solve CNF expressing E + PSP + PS only using variables corresponding to the 81 profiles and the ones obtained from them by permuting the $n = 6$ agents (=870.000 variables)
4. Extract MUS

The main takeaways

- Linear Programming can be an alternative to SAT-solving when working with non-discrete values
- Discretization might require to weaken axioms, but we obtain an even stronger result
- Reduction of problem by first obtaining impossibility when assuming some property which reduces number of distinct profiles, and then extending the impossibility when dropping the additional axiom