Homework #1

Deadline: Tuesday, 5 October 2010, 11:00

Question 1 (10 marks)
In analogy to the definition of Condorcet winners, a Condorcet loser is a candidate that would lose against any other candidate in a pairwise contest.

(a) Give an example that shows that the plurality rule can elect a Condorcet loser.

(b) Prove that the Borda rule never elects a Condorcet loser.

Remark: It is in fact possible to show that the Borda rule is the only positional scoring rule (with a strictly descending scoring vector) that satisfies this property.

Question 2 (10 marks)
In the late 1980s, Hervé Moulin published a paper showing that every voting procedure that satisfies the Condorcet Principle allows for situations where a voter has an incentive to not vote at all rather than to vote sincerely. Track down the paper in question, give a precise statement of the result mentioned above, and write up a succinct proof, using the notation and terminology from the course slides where applicable.

Question 3 (10 marks)
Analyse the computational complexity of the winner determination problem for the system of Single Transferable Vote (STV).

Bonus Question (20 marks)
In 1876, Charles Lutwidge Dodgson (better known as Lewis Carroll, the author of Alice in Wonderland) proposed the following voting procedure. Each voter strictly ranks all alternatives. The score of candidate $x$ is the minimal number of swaps of adjacent alternatives in a voter’s ranking required to make $x$ a Condorcet winner. The alternative(s) with the lowest score win(s). Implement the Dodgson rule. You may use any programming language you wish; the only constraint is that your program must run on the standard Linux environment provided by the FNWI. Your solution should be well documented: describe your algorithm and how you addressed the challenge of implementing a computationally intractable rule; provide comments on the code itself; clearly explain how to run your program; provide some examples that can be used for testing; and include an evaluation of the performance of your program. Your program should take as input a text file using a syntax as in the following example (which represents a ballot profile for three voters and four alternatives):

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a1 > a3 > a2 > a4
a1 > a4 > a3 > a2
a2 > a3 > a1 > a4
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Note: I will accept solutions for this question until 1 December 2010.