

Homework #7

Deadline: Monday, 16 December 2013, 17:00
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Question 1 (10 marks)

Recall the distributed approach to the allocation of indivisible goods discussed in class. Suppose all agents have non-negative *additive* valuation functions and they all assign value 1 to the full set of goods. Let A_0 be the initial allocation. Suppose that before negotiation starts we ask each agent to make an *initial downpayment* of $v_i(A_0) - \text{SW}_{\text{util}}(A_0)/n$ (note that this may be a negative number). That is, each agent pays an amount equal to their valuation for the bundle they received, and they then get back an amount equal to the average valuation of all agents. Then suppose the agents negotiate a sequence of *individually rational* deals involving just a *single item* at a time; and they continue until no more such deals are possible. Prove that if we enforce the use of the *globally uniform payment function*, then this sequence will culminate in an allocation that is *proportional*.

Question 2 (10 marks)

Describe a discrete procedure for dividing a cake between four agents that guarantees that each agent believes they received at least $1/6$ of the cake and that uses only three cuts. (Additional marking queries as well as moving knives are not allowed.)

(Adapted from J. Robertson and W. Webb, *Cake-Cutting Algorithms*, A.K. Peters, 1998.)