Computational Social Choice: Spring 2015

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Plan for Today

This will be the final lecture on judgment aggregation:

- *Truth-tracking* in judgment aggregation
- *Review* of material on judgment aggregation

Two Views of Judgment Aggregation

- *Perspective so far:* Agents give us their opinions, and we need to treat them in a "fair" manner, finding a collective judgment that accurately reflects the views of the group.
- *Epistemic perspective:* There is an objectively true judgment set (the *ground truth*) out there. Our agents perceive noisy signals and report them. We need to try and recover this ground truth.

The Condorcet Jury Theorem

This classical theorem applies to the case of JA with just a single pair of formulas in the agenda (i.e., a single binary issue):

Theorem 1 (Condorcet, 1785) *Suppose a jury of n voters need to select the better of two alternatives and each voter independently makes the correct decision with the same probability $p > \frac{1}{2}$. Then the probability that the majority rule returns the correct decision increases monotonically in n and approaches 1 as n goes to infinity.*

Proof sketch: By the law of large numbers, the number of voters making the correct choice approaches $p \cdot n > \frac{1}{2} \cdot n$. ✓

For a modern exposition, see Young (1995).

Writings of the Marquis de Condorcet. In I. McLean and A. Urken (eds.), *Classics of Social Choice*, University of Michigan Press, 1995.

H.P. Young. Optimal Voting Rules. *J. Economic Perspectives*, 9(1):51–64, 1995.

Generalisation: Unequal Expertise

Under the exact conditions of the Condorcet Jury Theorem and for *uniform priors* (*yes* and *no* are equally likely to be correct *a priori*) the *majority rule* is in fact *optimal:* no other rule is more likely to recover the ground truth correctly.

Some conditions can be relaxed: not all agents need to have the exact same expertise. The majority rule deciding correctly still approaches certainty as n increases, as long as the probabilities are "high enough" (but it is not necessarily optimal).

Unequal Expertise: Optimal Weights

Still just one issue (or several completely independent issues). But now suppose agents have *unequal expertise*. How to account for this?

Suppose p_i is the probability of agent i to be right on a given issue (same probability for all issues). What should i's weight w_i be?

We observe some agents say *yes* (N_{yes}) and some agents say *no* (N_{no}). Compare likelihoods of this observation for two scenarios:

correct answer is yes *correct answer is no*

$$\prod_{i \in N_{\text{yes}}} p_i \cdot \prod_{i \in N_{\text{no}}} (1 - p_i) \quad \geqslant^? \quad \prod_{i \in N_{\text{yes}}} (1 - p_i) \cdot \prod_{i \in N_{\text{no}}} p_i$$

Move N_{yes}-terms left, N_{no}-terms right, then apply logarithm:

$$\sum_{i \in N_{\text{yes}}} \log \frac{p_i}{1 - p_i} \quad \geqslant^? \quad \sum_{i \in N_{\text{no}}} \log \frac{p_i}{1 - p_i}$$

So: use weighted majority with *weights* $\log \frac{p_i}{1-p_i}$! (for uniform priors)

Several Issues: Estimating Accuracy

Suppose (again) p_i is the probability of agent i to be right on an issue (same probability for all issues). But now suppose *we do not know* p_i.

Idea: Try to *estimate* the *accuracy* p_i of agent i as her observed *agreement* with the majority rule.

$$p_i \;\approx\; \frac{\#\, \textit{issues where } i \textit{ and majority rule agree} + 0.5}{\#\, \textit{issues answered by } i + 1}$$

Thus: a possible rule would be to use weighted majority rule with weights $w_i = \log \frac{p_i}{1-p_i}$ for estimates of p_i's computed as above.

This rule has been used successfully for aggregating the judgments of non-expert workers in a crowdsourcing experiment on linguistic data.

C. Qing, U. Endriss, R. Fernández, and J. Kruger. Empirical Analysis of Aggregation Methods for Collective Annotation. Proc. COLING-2014.

Literature

To date, there has been precious little research on the epistemic approach to JA. All existing work makes strong limiting assumptions:

- Bozbay et al. (2014) focus on the special case of two logically independent issues.
- The other papers cited below all focus on very specific agendas (only one pair of compound formulas) and the premise-based rule.

I. Bozbay, F. Dietrich, and H. Peters. Judgment Aggregation in Search for the Truth. *Games and Economic Behavior*, 87:571–590, 2014.

L. Bovens and W. Rabinowicz. Democratic Answers to Complex Questions: An Epistemic Perspective. *Synthese*, 150(1):131–153, 2006.

S. Hartmann and J. Springer. Judgment Aggregation and the Problem of Tracking the Truth. *Synthese*, 187(1): 209–221, 2012.

G. de Clippel and K. Eliaz. Premise-Based versus Outcome-Based Information Aggregation. *Games and Economic Behavior*, 89:34–42, 2015.

Review: Judgment Aggregation

We have had nine lectures on judgment aggregation. A good part of the material is covered in the two expository papers cited below.

Next, we recall the main ideas . . .

C. List. The Theory of Judgment Aggregation: An Introductory Review. *Synthese*, 187(1):179–207, 2012.

U. Endriss. Judgment Aggregation. In F. Brandt, V. Conitzer, U. Endriss, J. Lang, and A.D. Procaccia (eds.), *Handbook of Computational Social Choice*. CUP, 2015.

Two Frameworks

We have used two closely related frameworks:

- *Formula-based judgment aggregation*
  - Need to accept/reject pairs of formulas in agenda
  - Rationality defined in terms of logical consistency
- *Binary aggregation with integrity constraints*
  - Need to accept/reject issues (no internal structure)
  - Rationality stated explicitly in terms of integrity constraints

Which is better depends on context and personal taste.

Both represent aggregation problems *compactly*, while, e.g., binary aggregation with explicitly specified rational outcomes does not.

Aggregation Rules

Some rules only introduced for one of the two frameworks, but the underlying ideas are all general:

- Majority rule, (uniform) quota rules
- Premise-based rule (with or without restrictions on premises)
- Conclusion-based rule (only discussed for the doctrinal paradox)
- Optimisation rules (all highly complex): max-sum rule (Kemeny), max-number rule (Slater), greedy-max rule (Tideman)
- Representative-voter rules: average-voter rule, majority-voter rule, plurality-voter rule (not covered: ranked-voter rule)

Only the latter two groups always guarantee *collective rationality*.

For the optimisation rules, we have been guided by similar rules for *preference aggregation* (but the ideas are more general than that).

Not discussed in any detail: how to *break ties*

Modelling

We have focused on JA in the abstract, but it can be used to model various forms of collective decision making:

- decision making in judicial courts
- coordination and decision making in multiagent systems
- collective annotation via crowdsourcing
- embedding of preference aggregation into JA is possible

The latter allows importing ideas from voting theory, a much more mature area of research.

Methodology: Philosophical Perspective

We have not emphasised this in the course, but each of the axioms constraining "reasonable" rules that we have considered can and should be argued for on normative grounds:

- nondictatoriality
- anonymity
- neutrality
- independence
- monotonicity
- unanimity
- strategy-proofness
- ...

Methodology: Mathematical Perspective

Our main mathematical tool has been the *axiomatic method*.

Types of results (about axioms only):

- *Characterisation* of rules: quota rules, majority rule

Types of results (about interaction with rationality requirements):

- *Impossibility* results: some combinations of axioms are impossible to achieve (for sufficiently rich agendas)
- *Safety* results (universal agenda characterisation): all rules satisfying certain axioms are collectively rational for sufficiently poor agendas
- Existential *agenda characterisation:* there exists a rule satisfying certain axioms that is collectively rational *iff* the agenda is sufficiently poor

Focus was on formula-based JA, but in principle, instead of agenda properties we could speak of classes of IC's (and we have: "lifting" of IC's).

Methodology: Computational Perspective

Considering aggregation as an algorithmic problem, we have analysed the computational complexity of determining the outcome for several rules ("*winner determination problem*"):

- Easy: quota rules (low polynomial)
- Hard: max-sum rule (complete for parallel access to NP)
- Depends: premise-based rule (depends on restrictions to agenda)

Also other problems arising in JA require algorithms:

- Checking *safety:* highly intractable (e.g., for the majority rule)
- Strategic *manipulation:* NP-hard for the premise-based rule
- Various forms of *bribery* and *control*

Complexity classes seen: P, NP, coNP, Θ_2^P, Π_2^P

Not discussed, but important: practical *algorithms* for all of this

Methodology: Game-Theoretical Perspective

Lecture on strategic behaviour hinted at connections to game theory and mechanism design:

- Agents may lie when reporting their judgment
- Questions of how to best model individual *preferences*
- *Strategy-proofness* possible, but rare for attractive rules

Not discussed (and not yet treated in the literature): strategic manipulation under *partial information*, *iterated manipulation*, ...

Methodology: Statistical Perspective

We have been very brief on this, but it is clear that in principle JA can be interpreted as a means for *truth-tracking* (epistemic approach).

To do so we can use statistical methods for estimating the most likely ground truth given the observed opinions of the agents.

Conclusion

This concludes the lectures on judgment aggregation.

- Young research area: while there have been a couple of precursors in the literature, the field is really only a little over a decade old.
  - Still possible to get a good global view of the field.
  - Clear opportunities to make original contributions yourself.
- Methods used in JA reflect use of methods more widely in computational social choice.

What next?

- Two lectures on topics other than JA
- Two meetings on how to write a paper + how to give a talk
- Individual research meetings with each group
- Final presentations by the groups